Parity Games with Weights

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and Martin Zimmermann (Saarland University)

Alexander Weinert

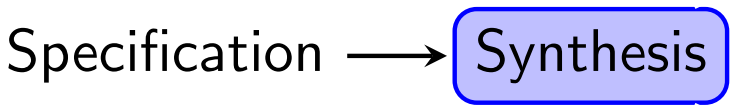
Saarland University

September 6th, 2018

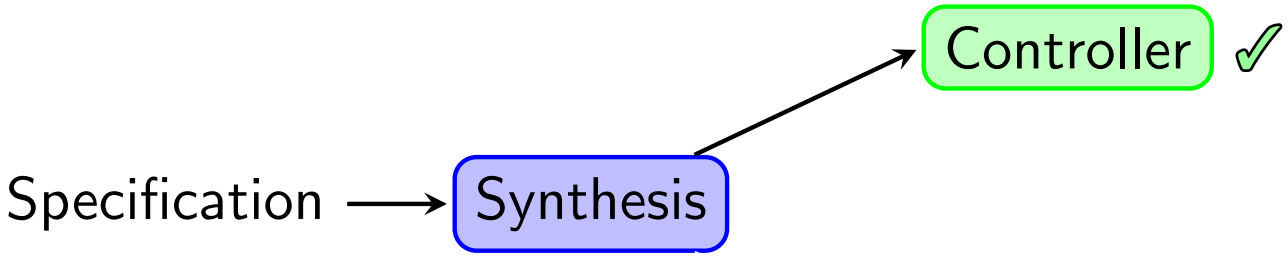
Controller Synthesis

Specification

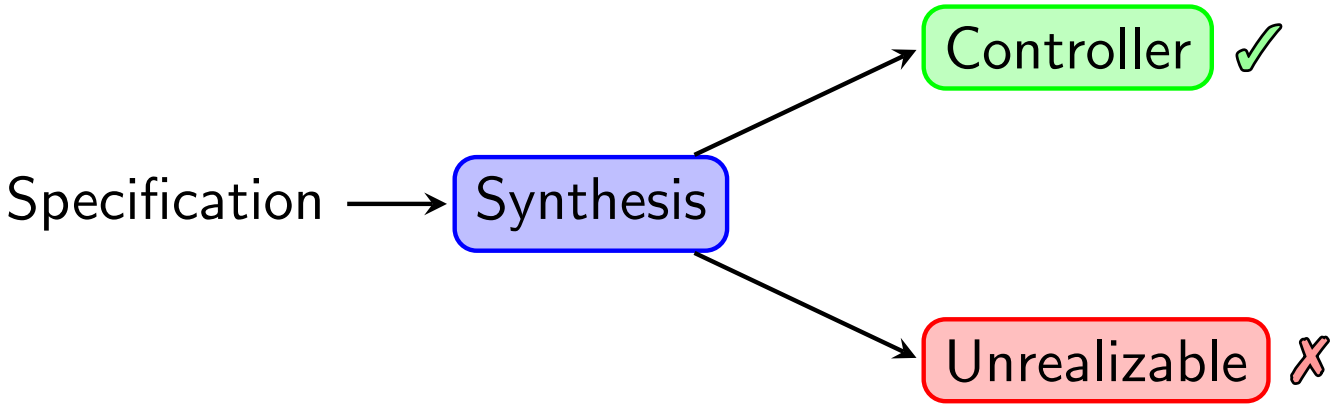
Controller Synthesis



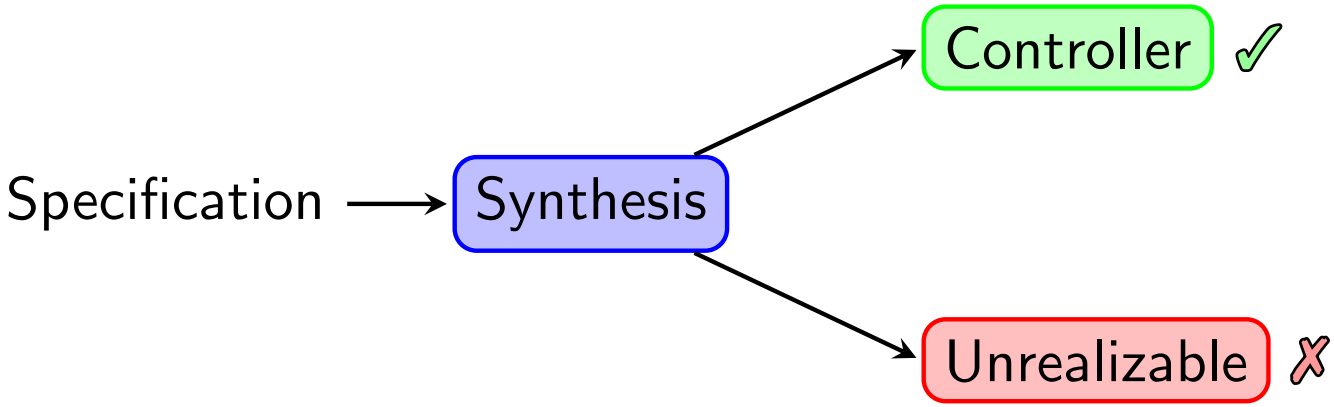
Controller Synthesis



Controller Synthesis



Controller Synthesis



In most cases boils down to solving parity game

Parity Games

Parity Games

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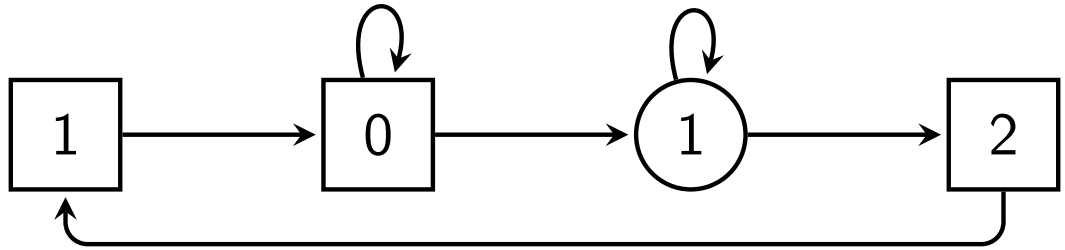
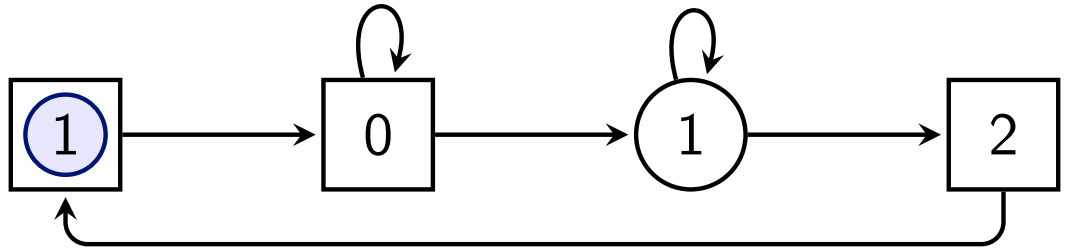


Parity Games



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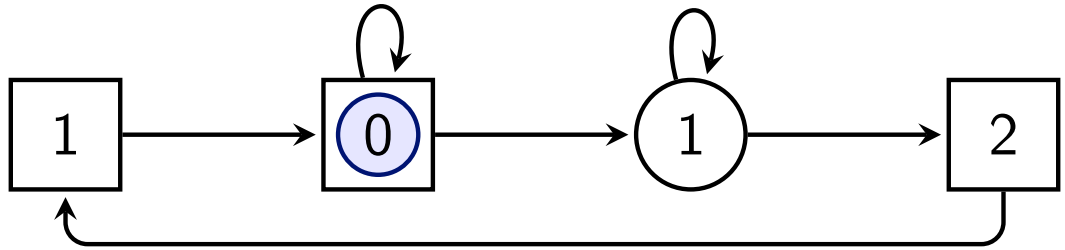
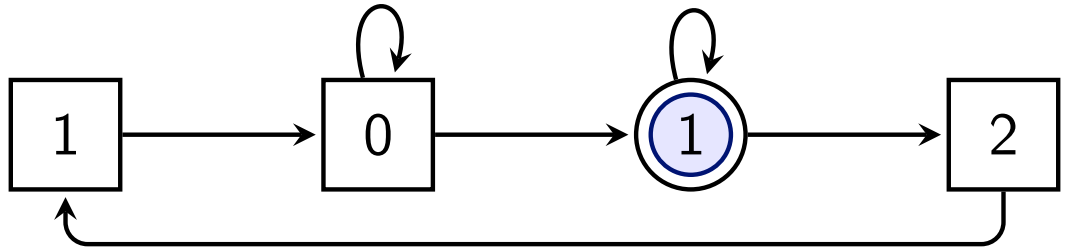
Parity Games



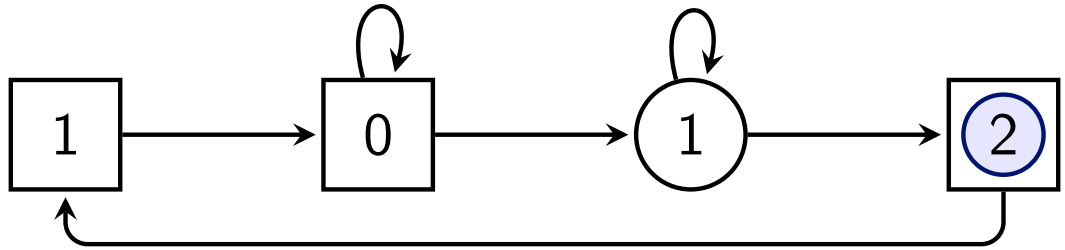
$1 \longrightarrow 0$

Parity Games



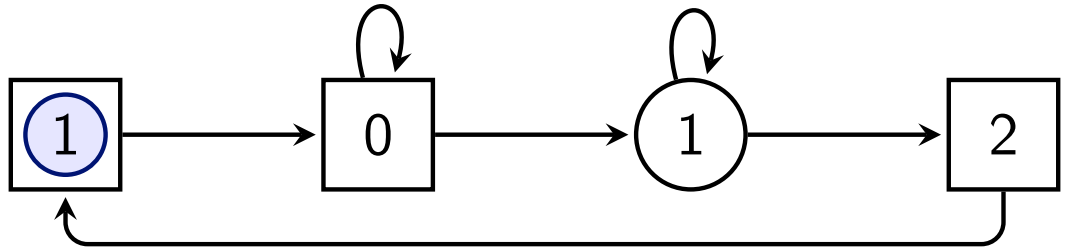
$1 \longrightarrow 0 \longrightarrow 1$

Parity Games



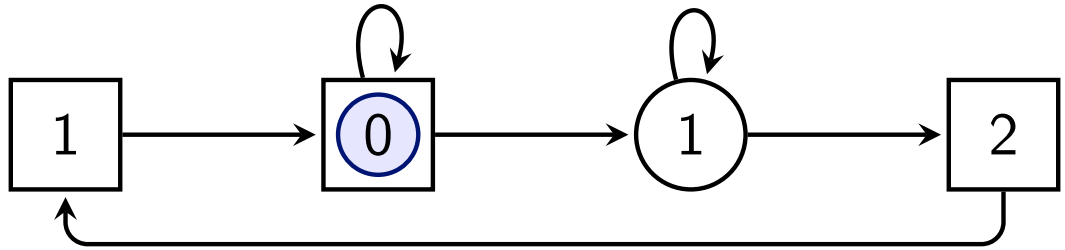
$1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2$

Parity Games



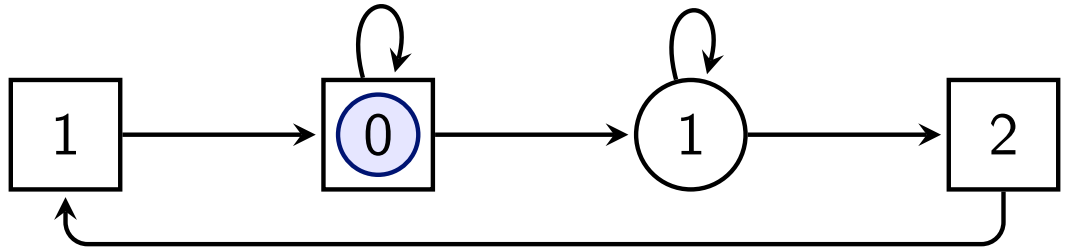
$1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2 \longrightarrow 1$

Parity Games



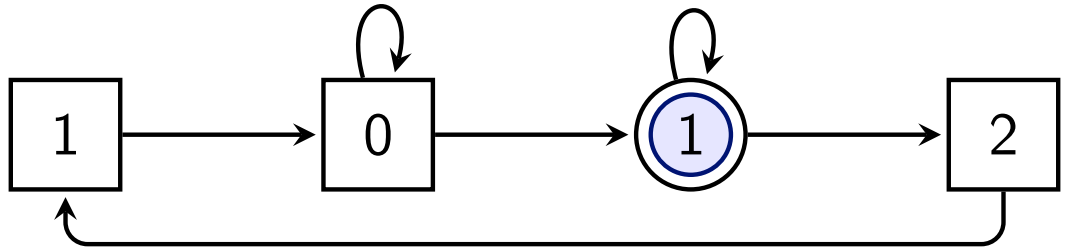
$1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2 \longrightarrow 1 \longrightarrow 0$

Parity Games



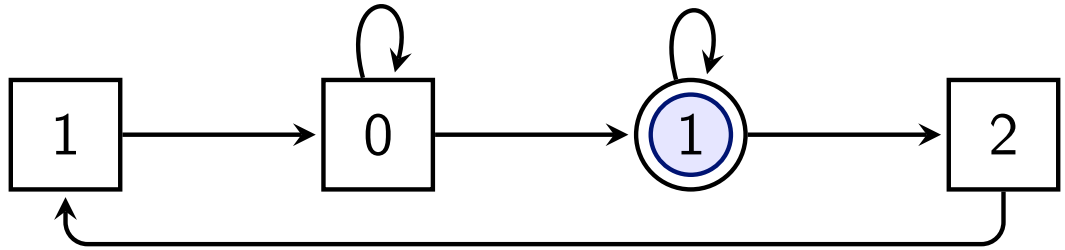
$1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0$

Parity Games



$1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 1$

Parity Games



$1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 1 \longrightarrow 1$

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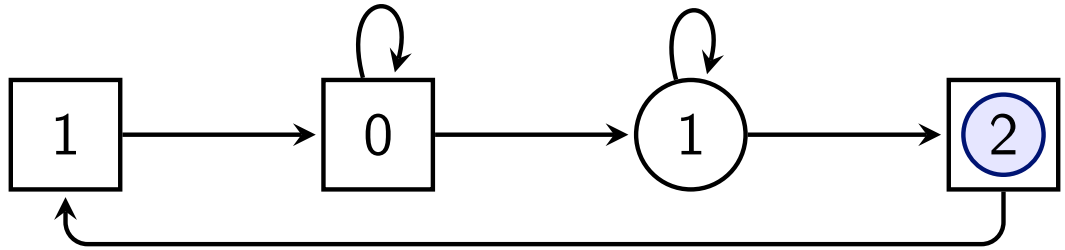
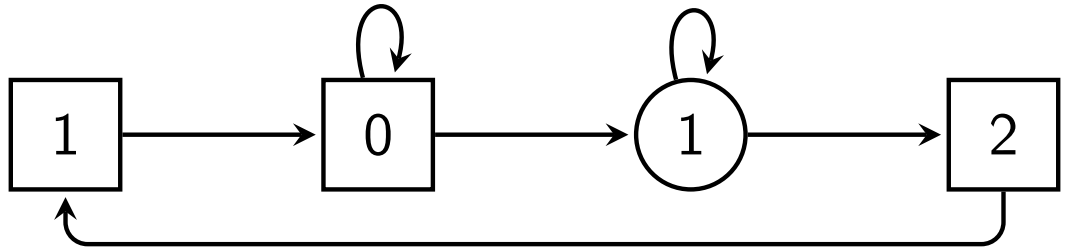


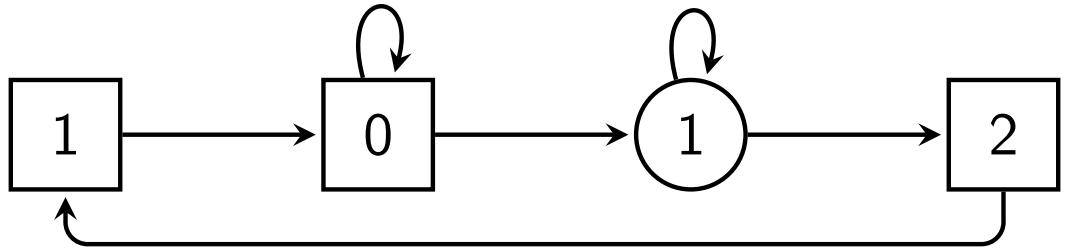
$$1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 1 \longrightarrow 1 \longrightarrow 2 \longrightarrow \cdots$$

Parity Games



$$1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 1 \longrightarrow 1 \longrightarrow 2 \longrightarrow \cdots$$

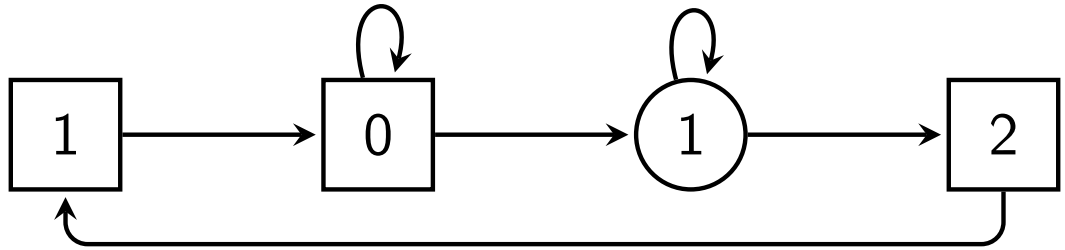
Parity Games



$1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 1 \longrightarrow 1 \longrightarrow 2 \longrightarrow \cdots$

Max. color inftly often even

Parity Games



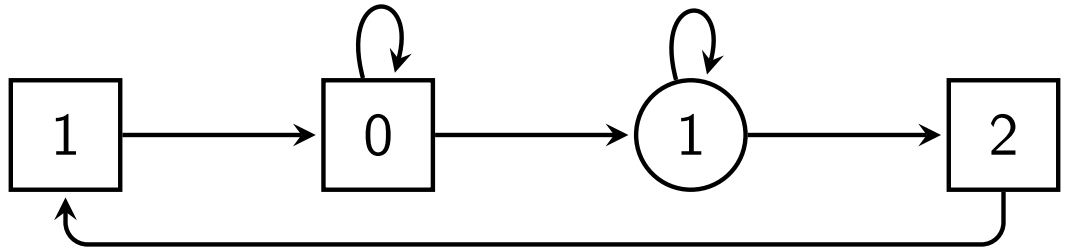
$1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 1 \longrightarrow 1 \longrightarrow 2 \rightarrow \cdots$

Max. color inftly often even

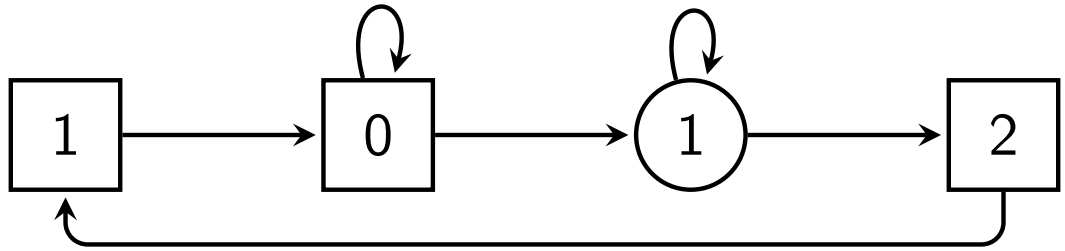
Proposition (Jurdziński 1998, Calude et al. 2017)

Solving parity games is in $\mathrm{UP} \cap \mathrm{co\text{-}UP}$ *and they can be solved in quasi-polynomial time. Both players have memoryless winning strategies.*

Finitary Parity Games

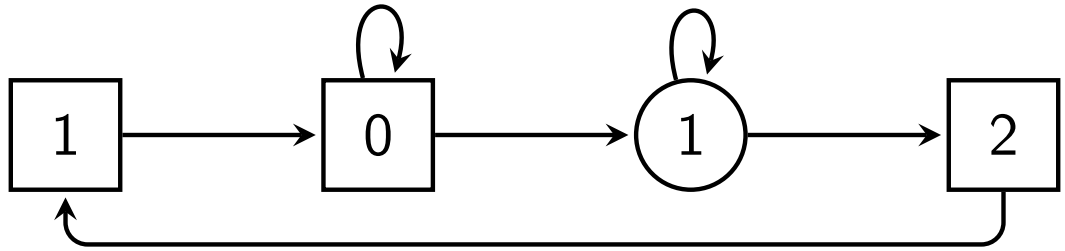


Finitary Parity Games



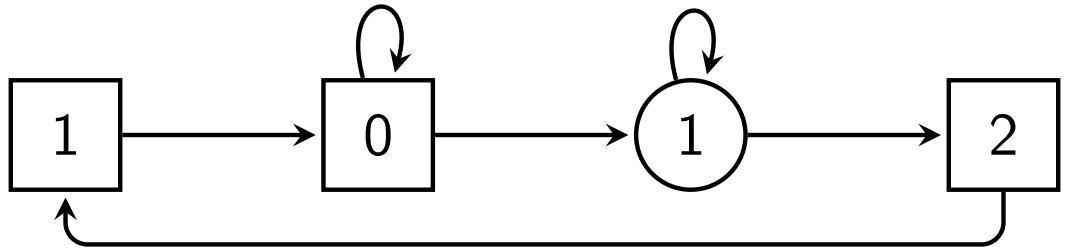
$$1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 1 \longrightarrow 1 \longrightarrow 2 \longrightarrow \cdots$$

Finitary Parity Games



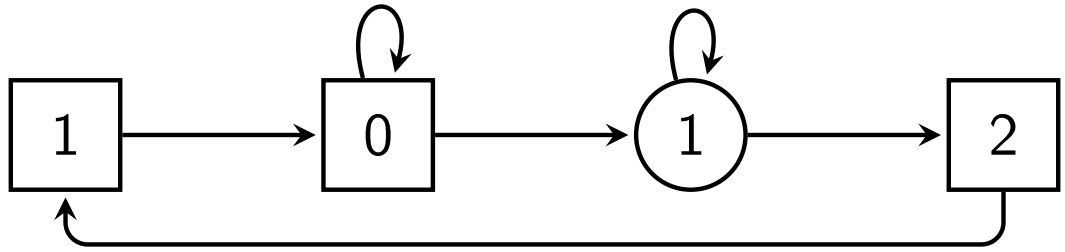
$$\underbrace{1 \to 0 \to 1 \to 2}_{3 \text{ steps}} \to 1 \to 0 \to 0 \to 1 \to 1 \to 2 \to \cdots$$

Finitary Parity Games



$$\underbrace{1 \to 0 \to 1 \to 2}_{3 \text{ steps}} \to \underbrace{1 \to 0 \to 0 \to 1 \to 1 \to 2}_{5 \text{ steps}} \to \cdots$$

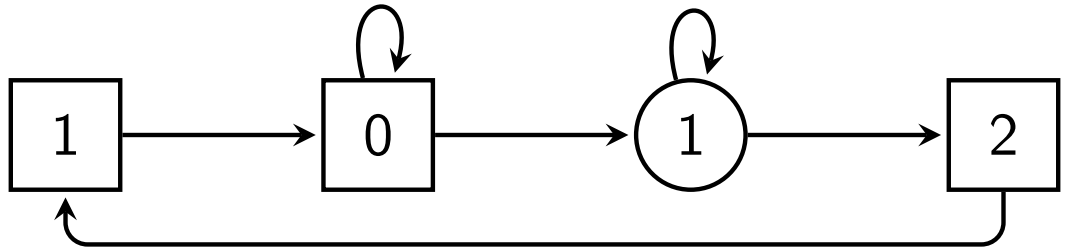
Finitary Parity Games



$$\underbrace{1 \to 0 \to 1 \to 2}_{3 \text{ steps}} \to \underbrace{1 \to 0 \to 0 \to 1 \to 1 \to 2}_{5 \text{ steps}} \to \cdots$$

Aim for Player 0:
Eventually bound steps between requests and answers

Finitary Parity Games



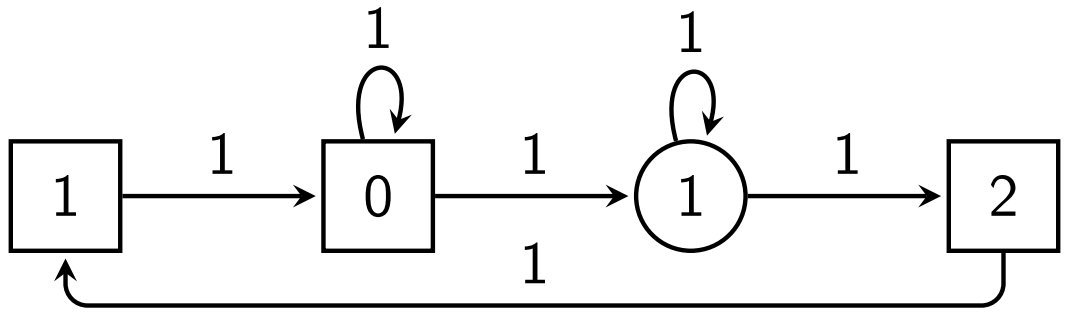
$$\underbrace{1 \to 0 \to 1 \to 2}_{3 \text{ steps}} \to \underbrace{1 \to 0 \to 0 \to 1 \to 1 \to 2}_{5 \text{ steps}} \to \cdots$$

Aim for Player 0:
Eventually bound steps between requests and answers

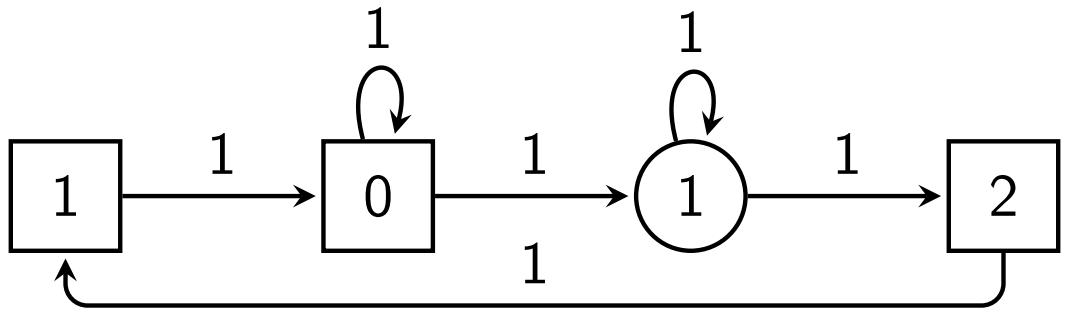
Proposition (Chatterjee, Henzinger, and Horn, 2009)

Solving finitary parity games is in PTIME. *Player* 0 *has memoryless winning strategies.*

Parity Games with Costs

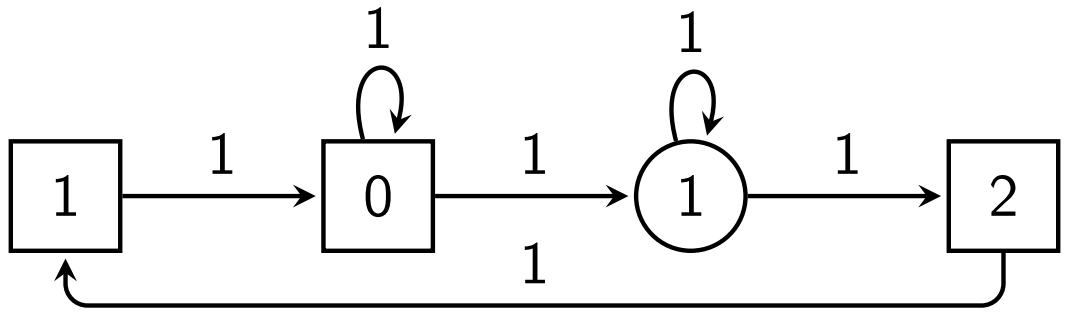


Parity Games with Costs



$$1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 1 \longrightarrow 1 \longrightarrow 2 \longrightarrow \cdots$$

Parity Games with Costs



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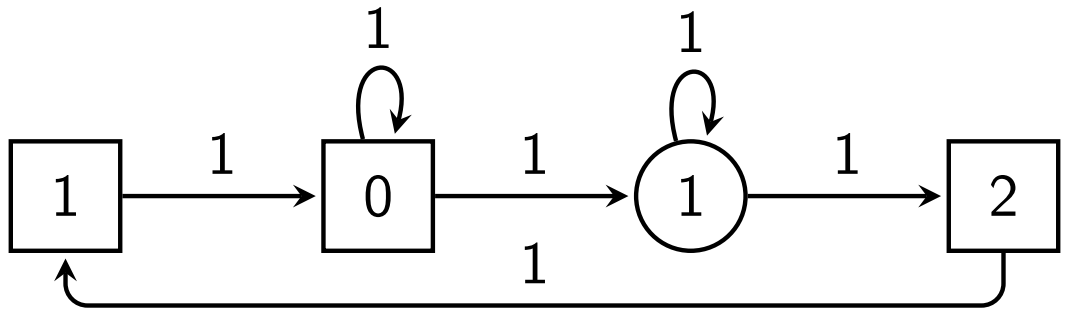


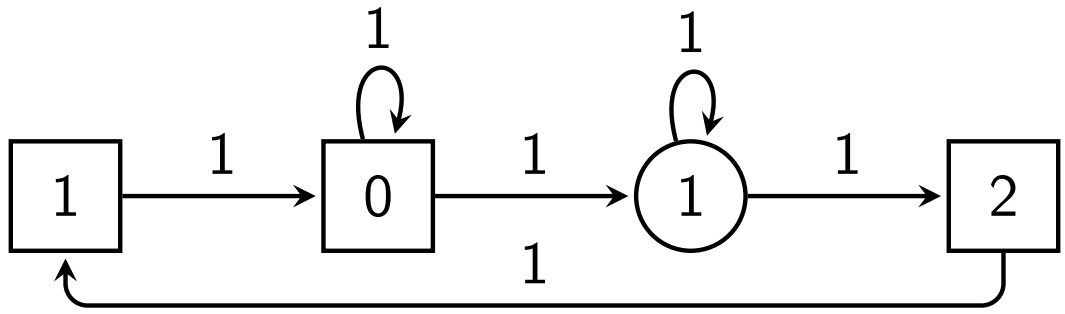
$1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 1 \longrightarrow 1 \longrightarrow 2 \longrightarrow \cdots$

Weight 3

Aim for Player 0:

Eventually bound steps between request and answer

Parity Games with Costs



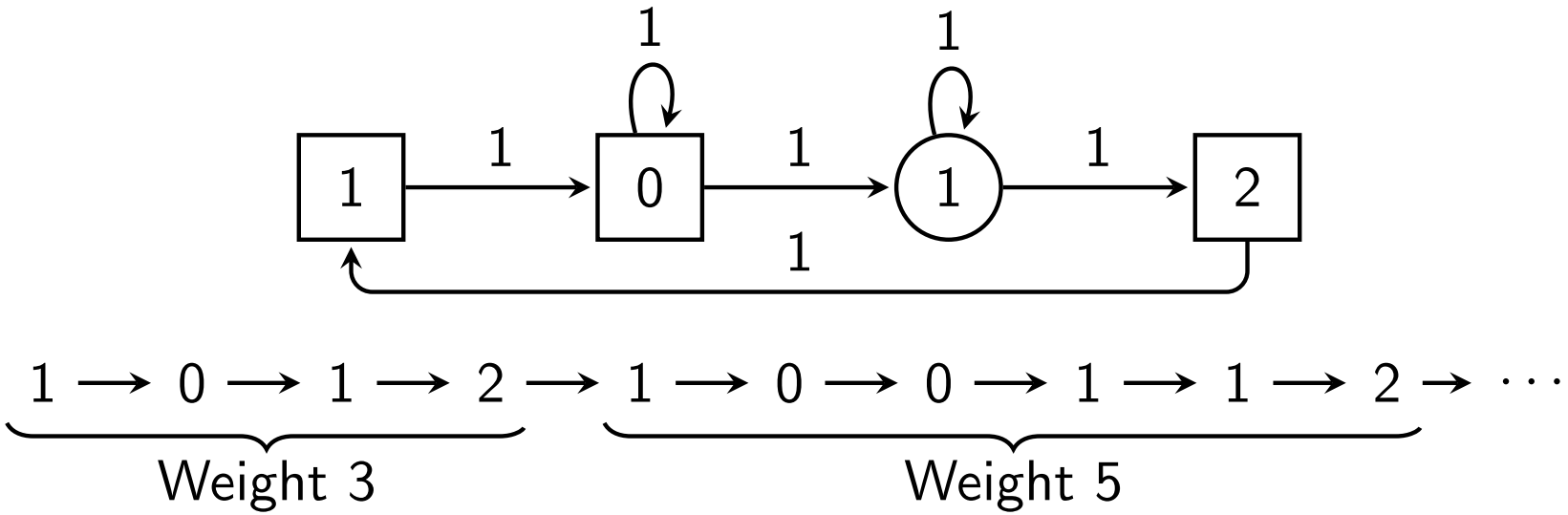
$1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 1 \longrightarrow 1 \longrightarrow 2 \longrightarrow \cdots$

Weight 3 Weight 5

Aim for Player 0:

Eventually bound steps between request and answer

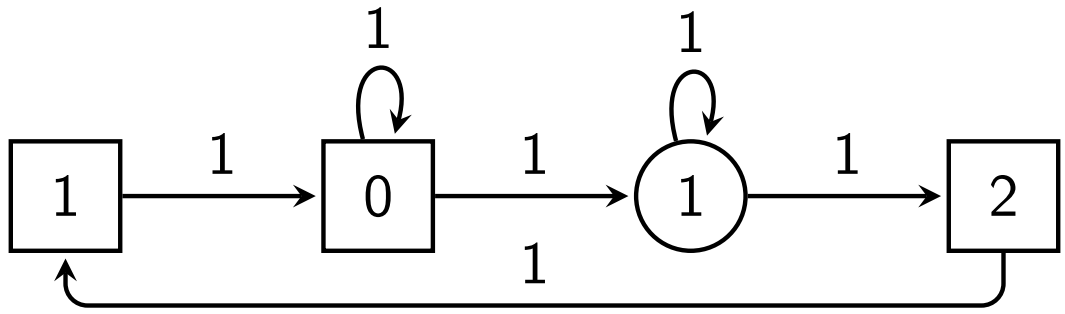
Parity Games with Costs



Aim for Player 0:

Eventually bound ~~steps~~ weight between request and answer

Parity Games with Costs



$1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 1 \longrightarrow 1 \longrightarrow 2 \longrightarrow \cdots$

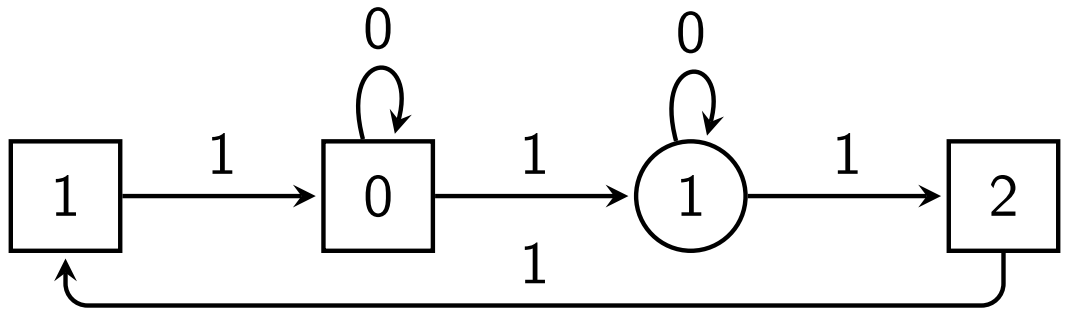
Weight 3 Weight 5

Aim for Player 0:
Eventually bound ~~steps~~ weight between request and answer

Proposition (Fijalkow and Zimmermann 2014 / Mogavero, Murano, and Sorrentino 2015)

Solving parity games with costs is in $\mathrm{UP} \cap \text{co-}\mathrm{UP}$. *Player* 0 *has memoryless winning strategies.*

Parity Games with Costs



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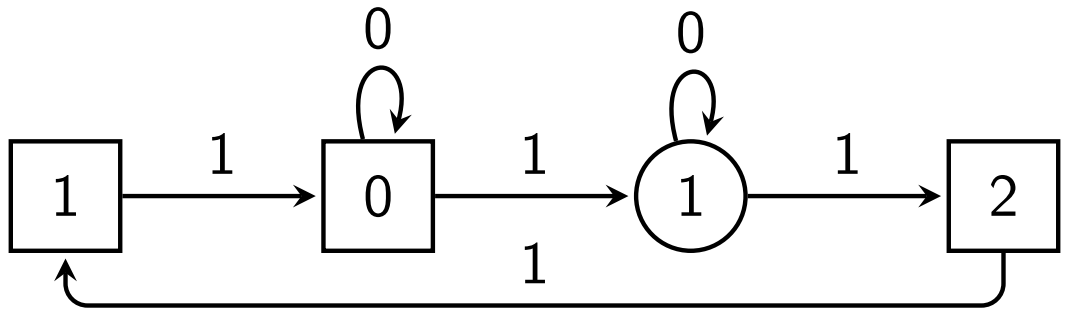

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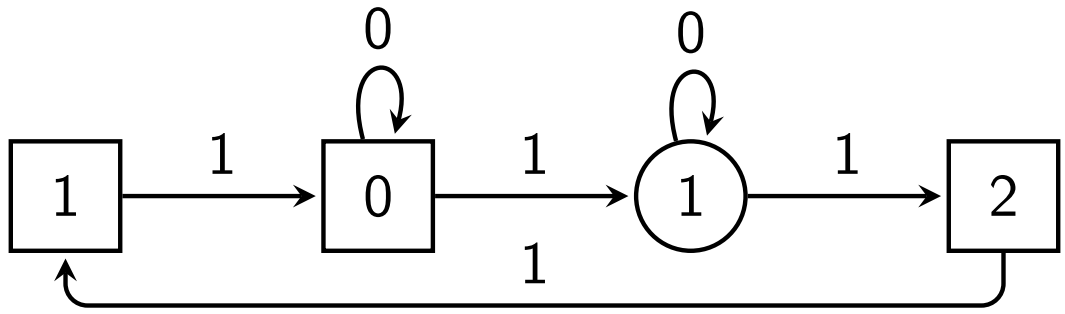
Parity Games with Costs



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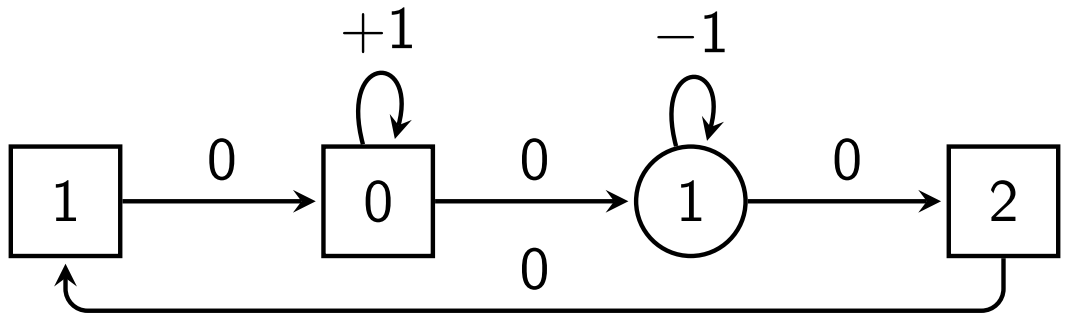
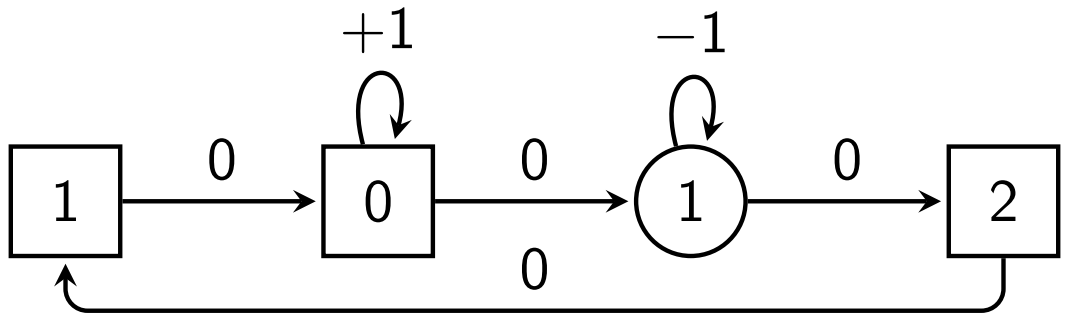
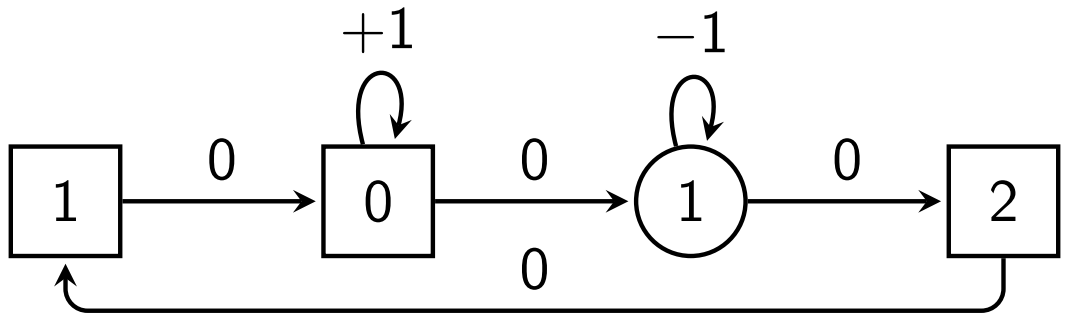


Parity Games with Weights



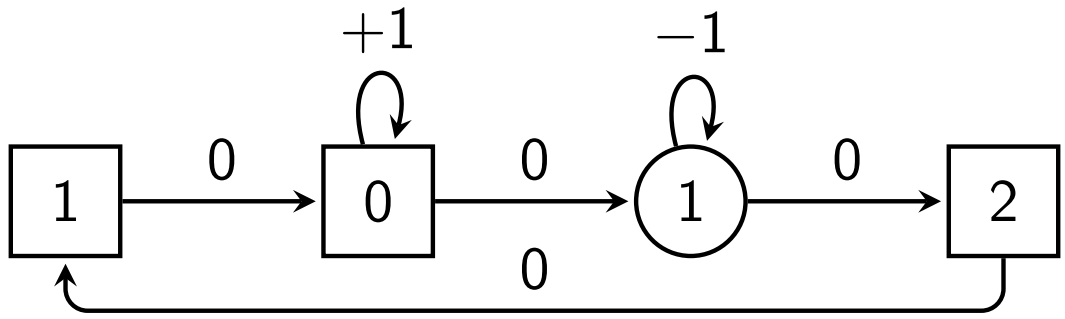
$$1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 1 \longrightarrow 1 \longrightarrow 2 \longrightarrow \cdots$$

Parity Games with Weights



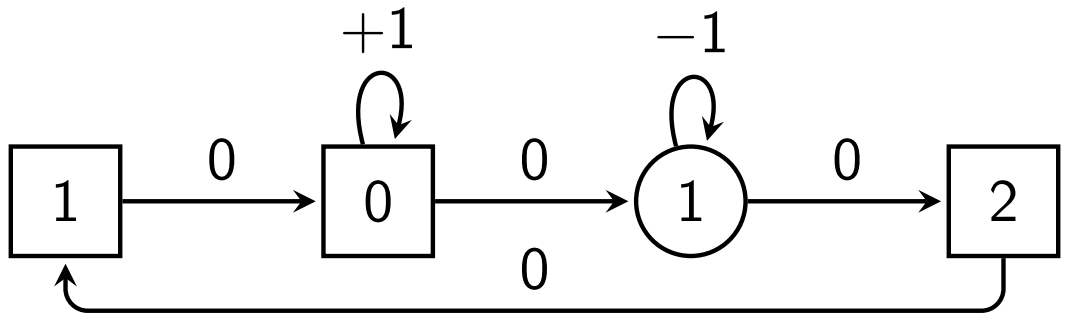
$$\underbrace{1 \to 0 \to 1 \to 2}_{\text{Weight } 0} \to 1 \to 0 \to 0 \to 1 \to 1 \to 2 \to \cdots$$

Parity Games with Weights



$$\underbrace{1 \to 0 \to 1 \to 2}_{\text{Weight } 0} \to \underbrace{1 \to 0 \to 0 \to 1 \to 1 \to 2}_{\text{Weight } 0} \to \cdots$$

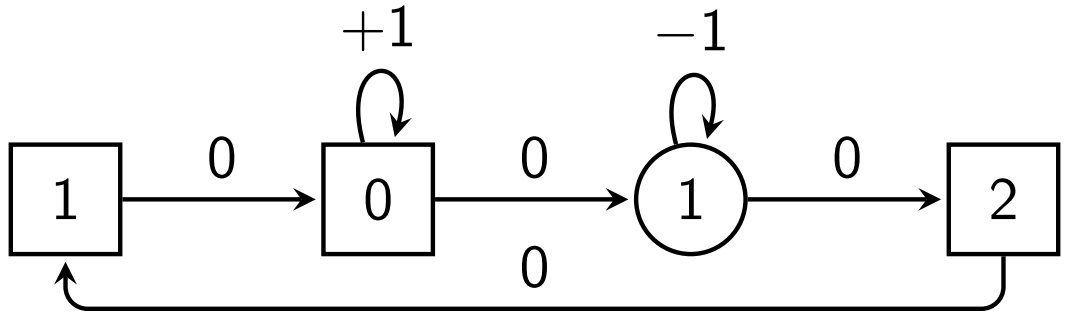
Parity Games with Weights



$$\underbrace{1 \to 0 \to 1 \to 2}_{\text{Weight } 0} \to \underbrace{1 \to 0 \to 0 \to 1 \to 1 \to 2}_{\text{Weight } 0} \to \cdots$$

Not the complete picture...

Cost Measure



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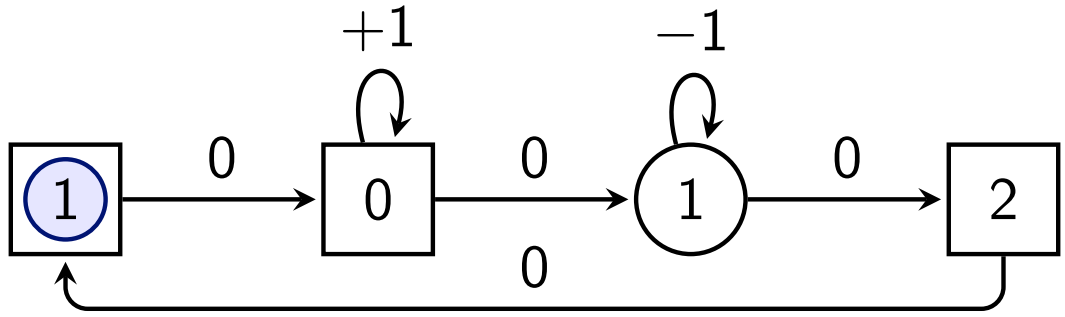

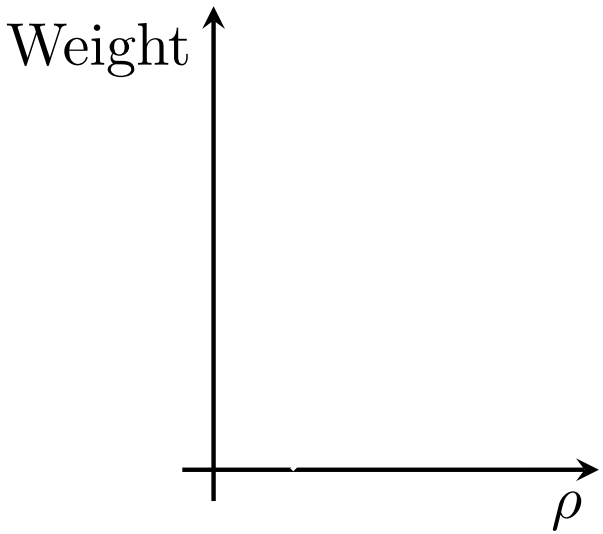

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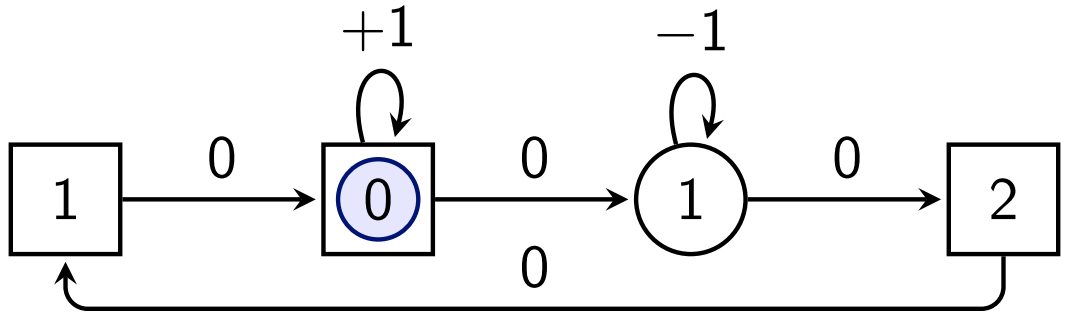

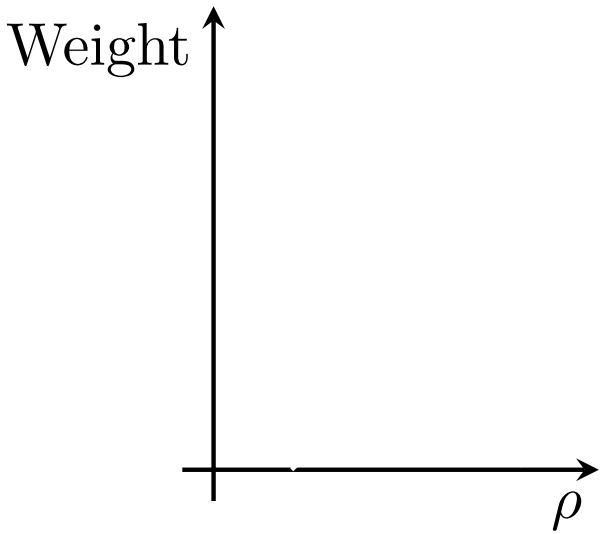

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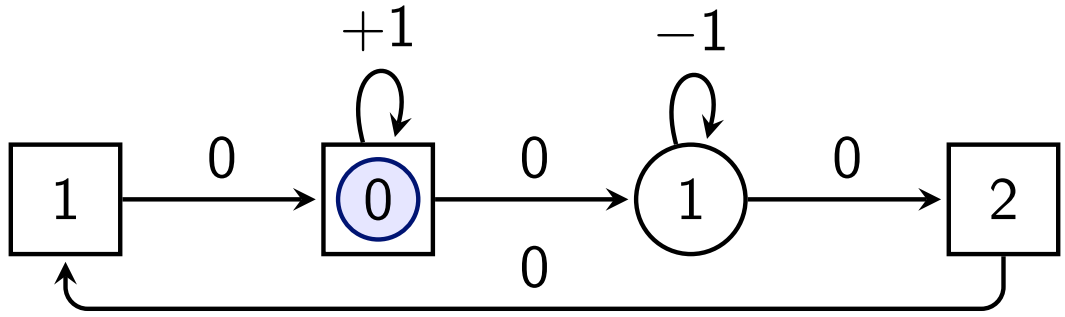

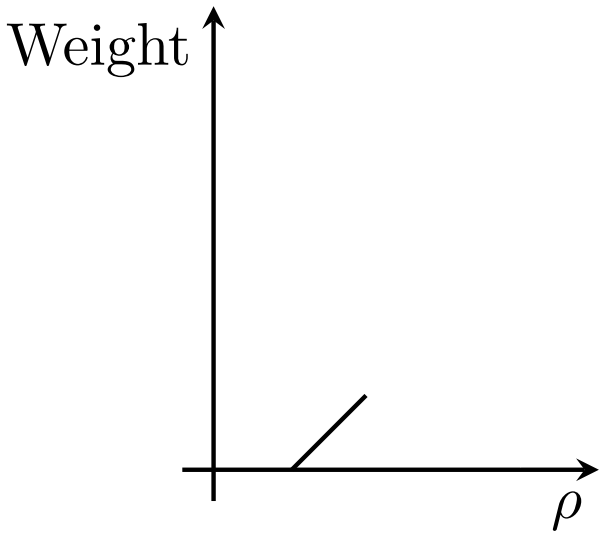

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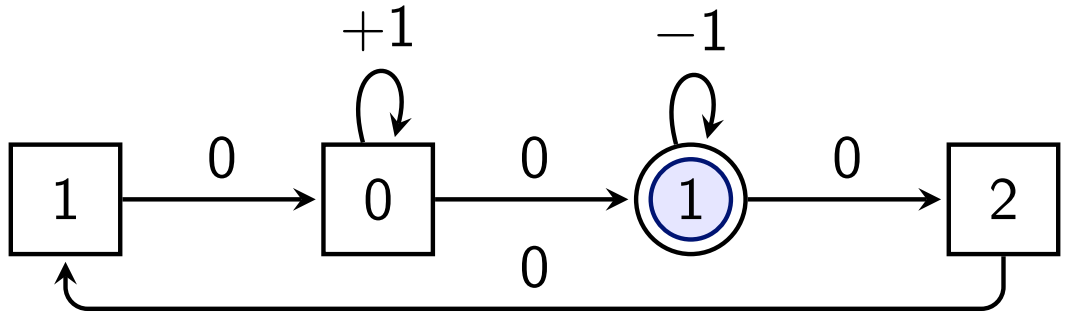
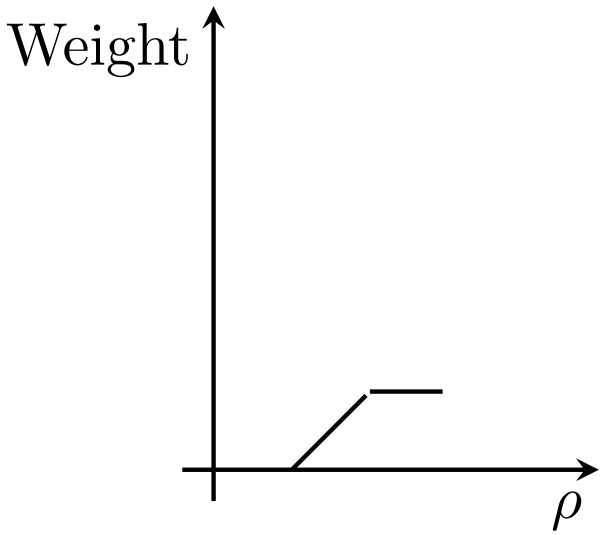
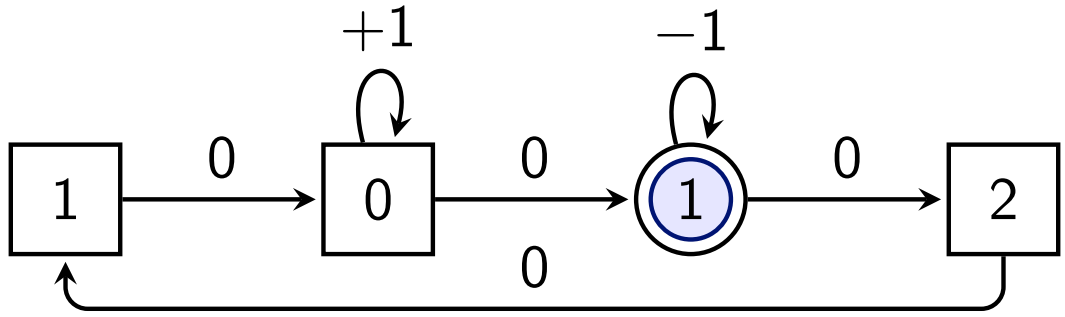
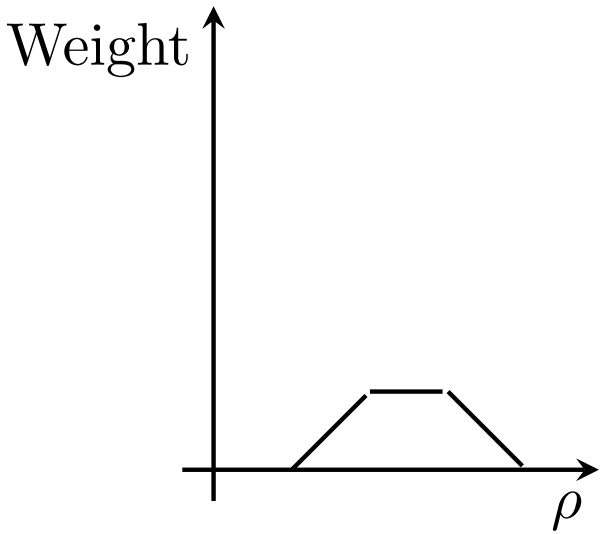




Cost Measure





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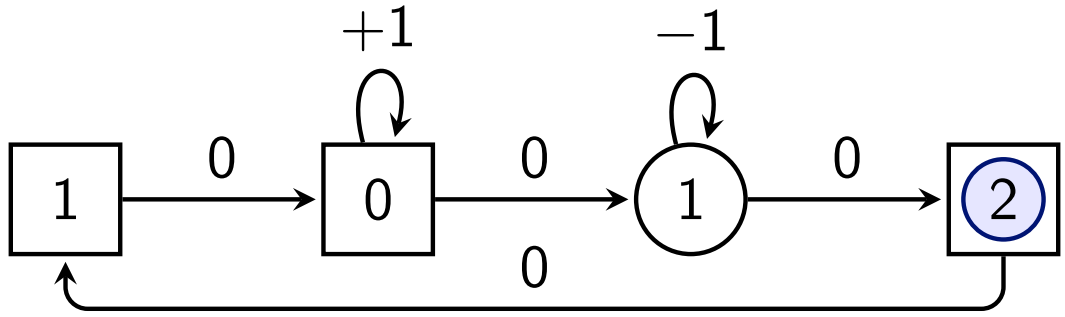

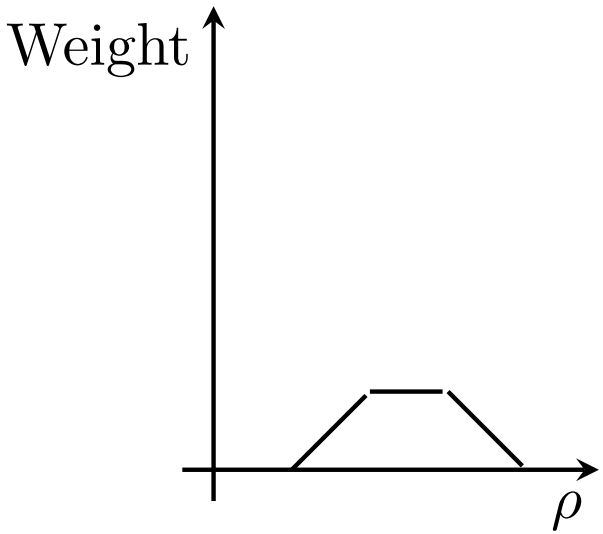

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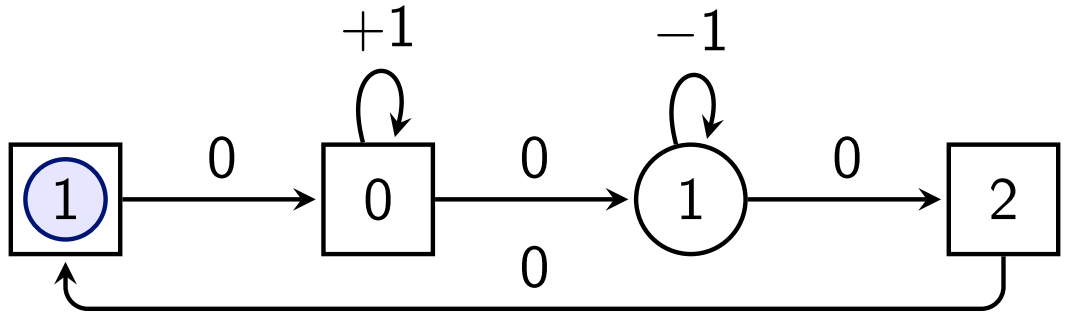

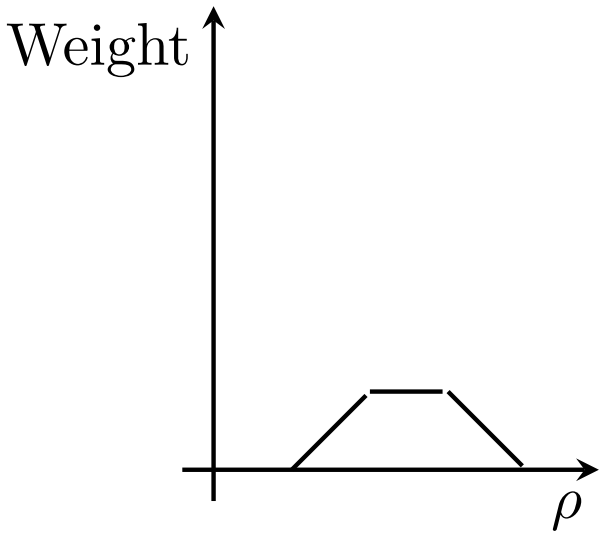

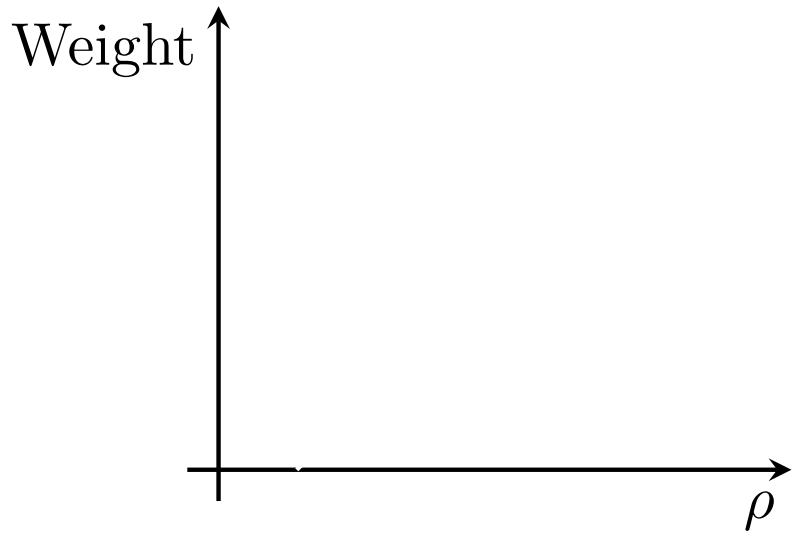

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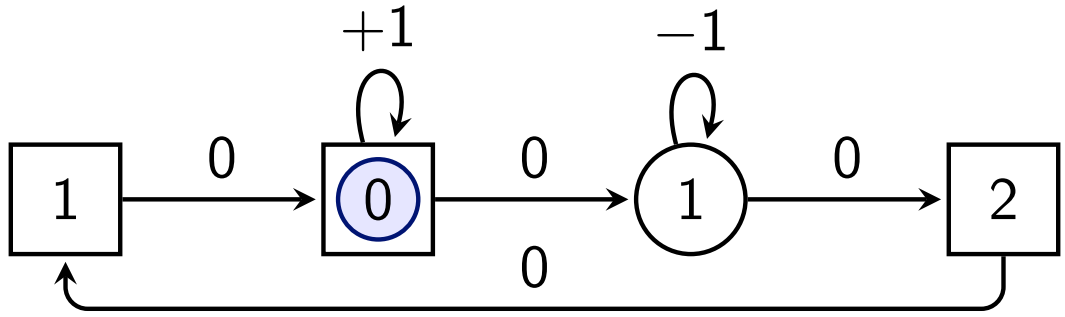

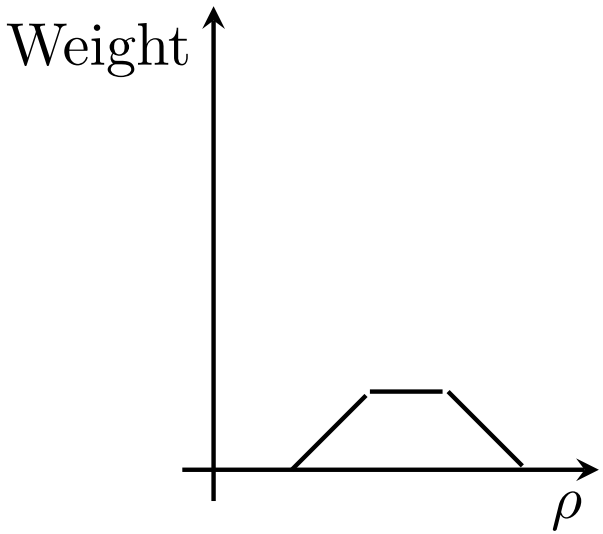

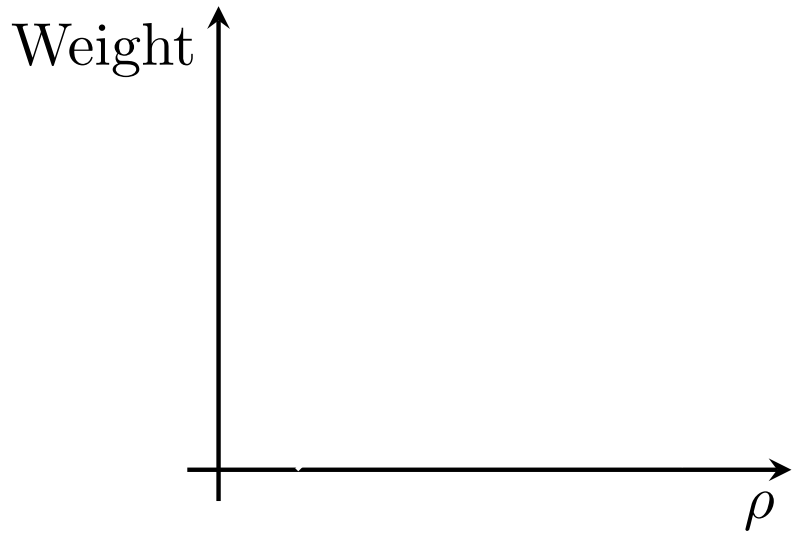

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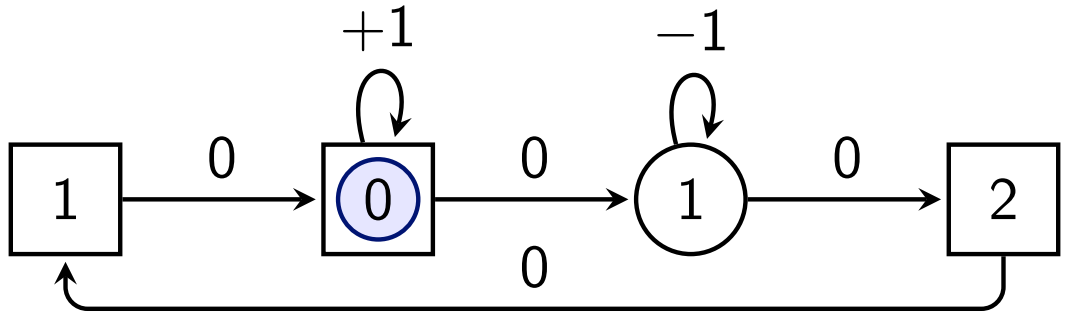

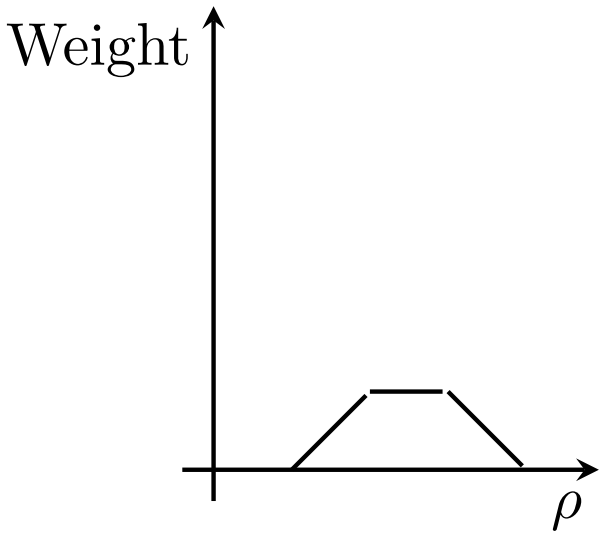

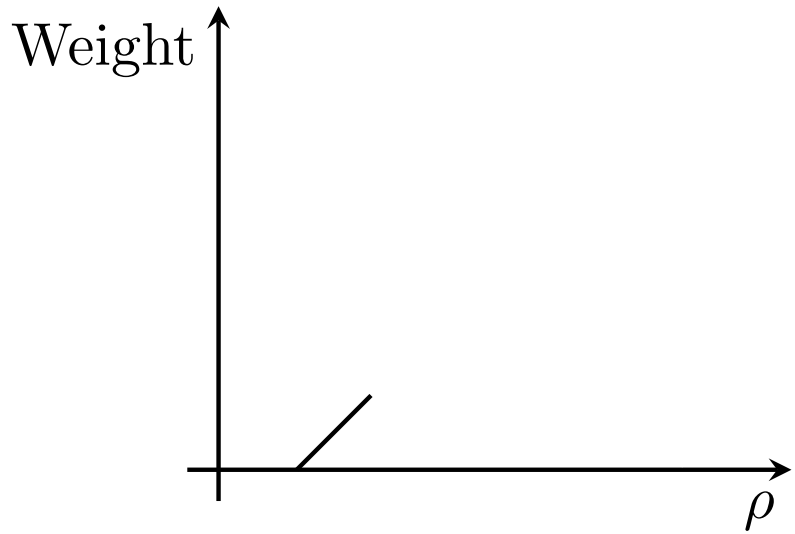

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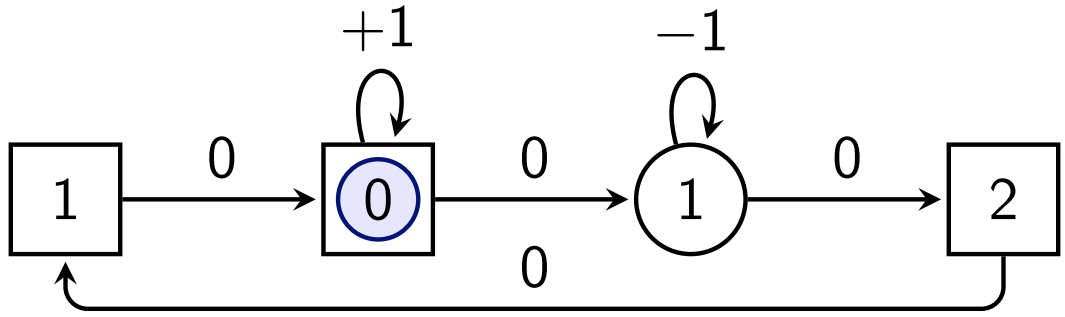

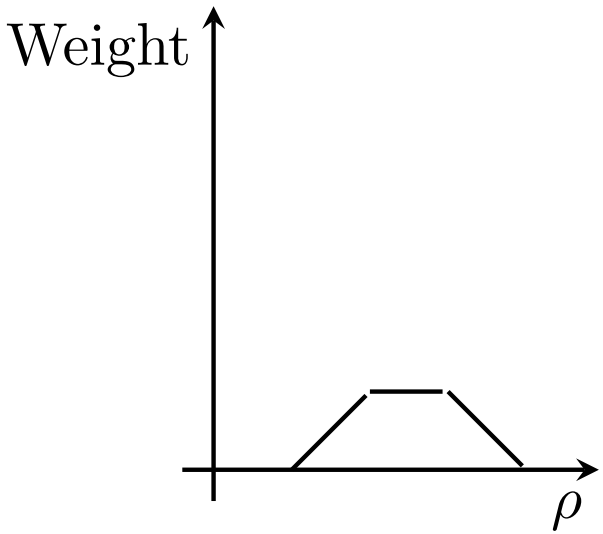

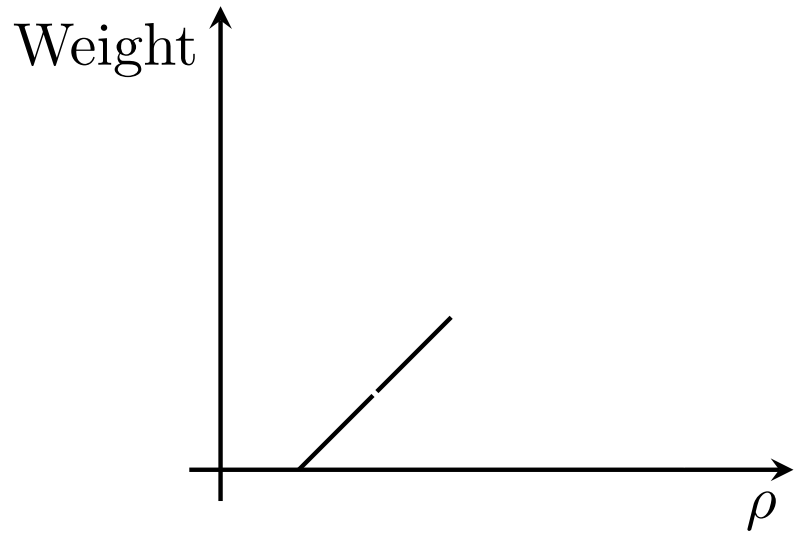

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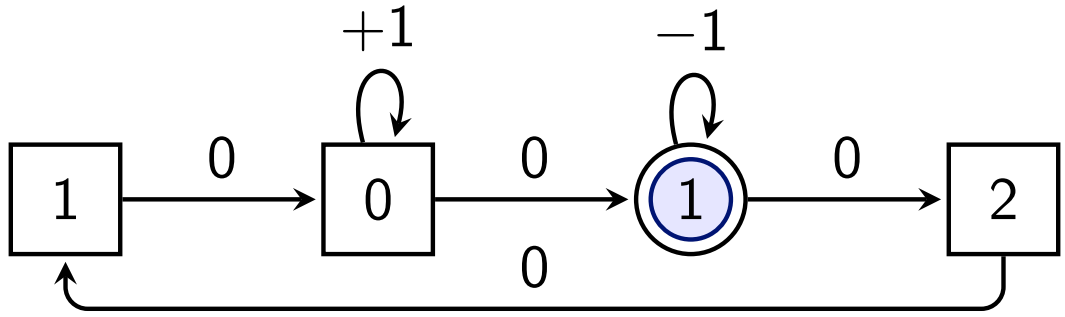

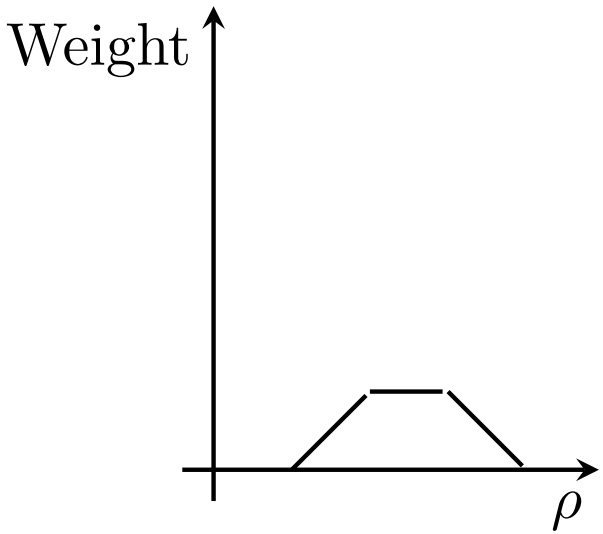

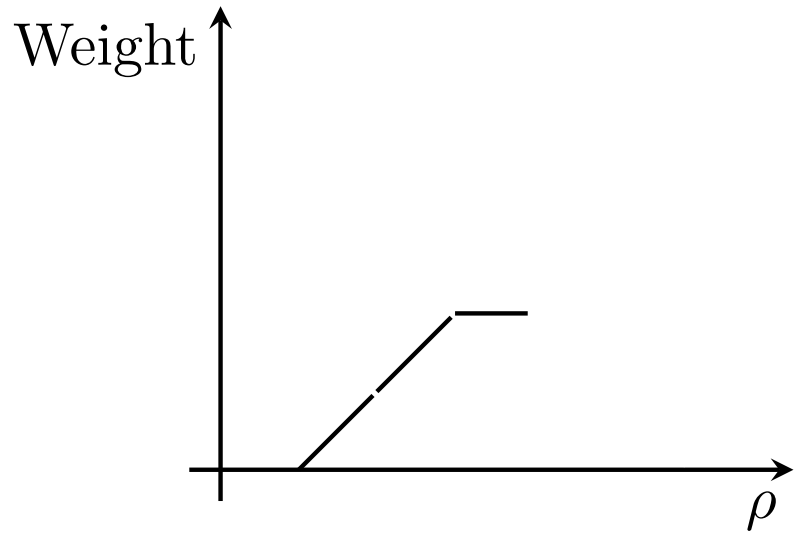

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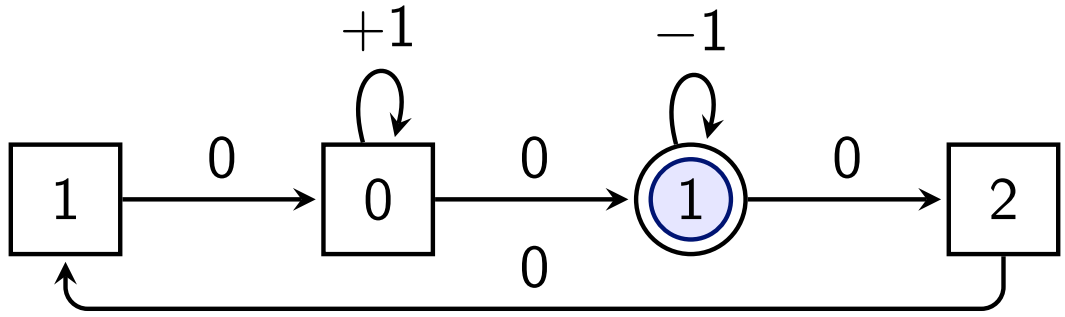

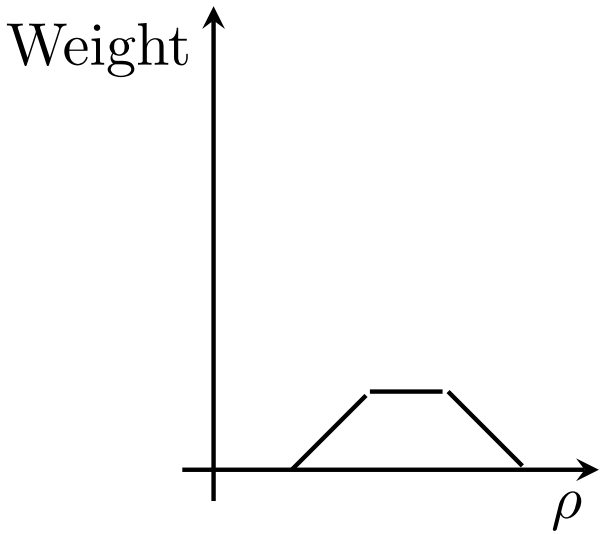

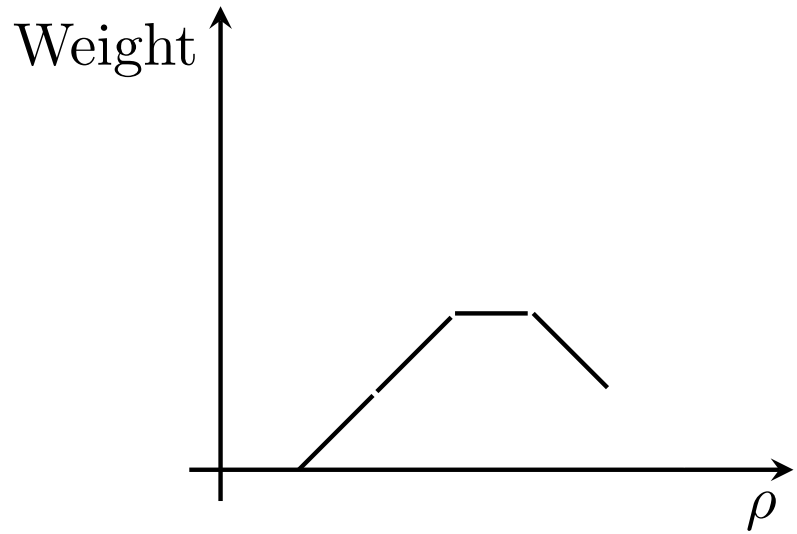

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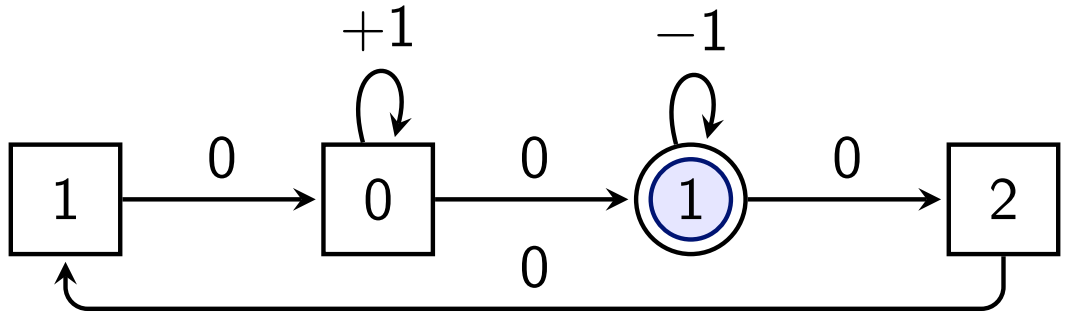

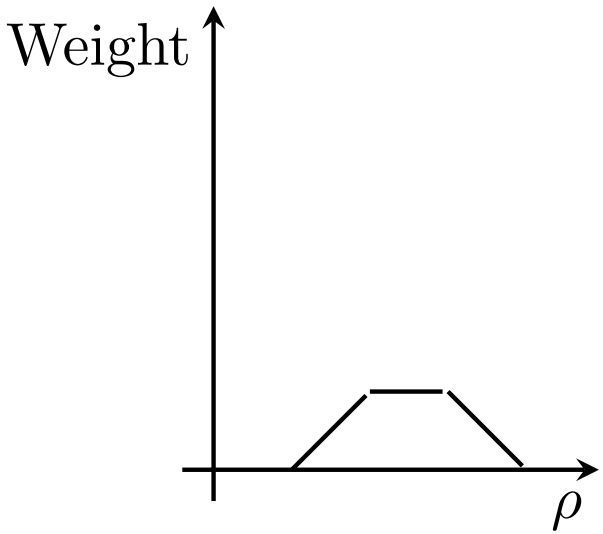

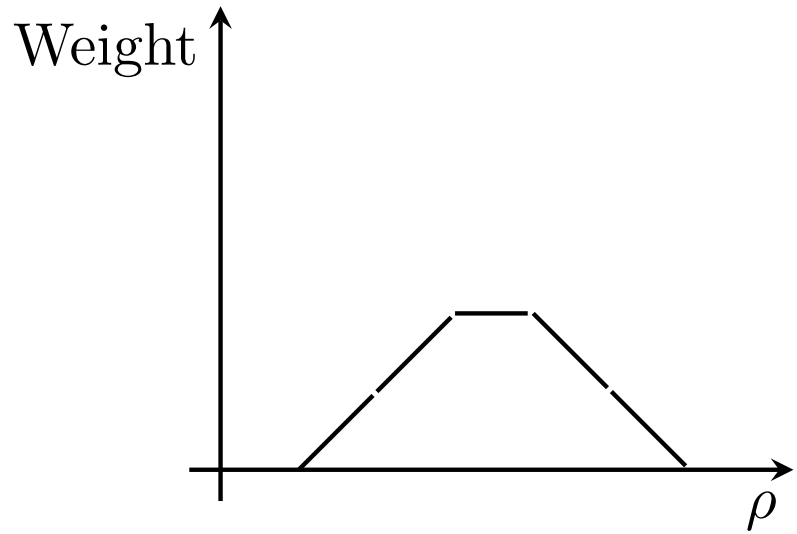

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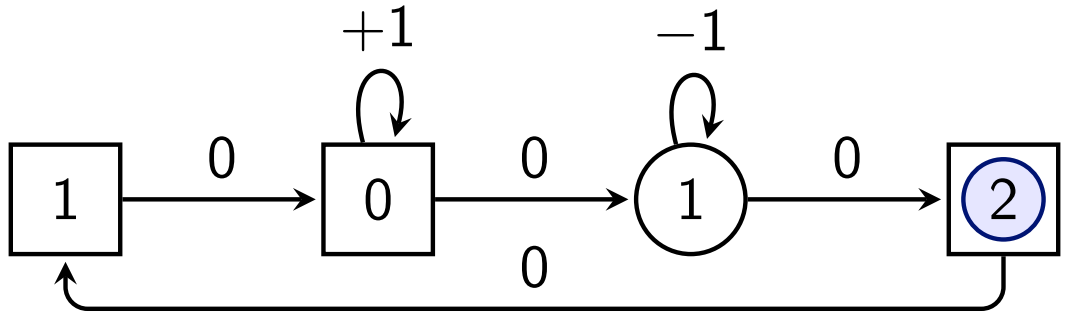
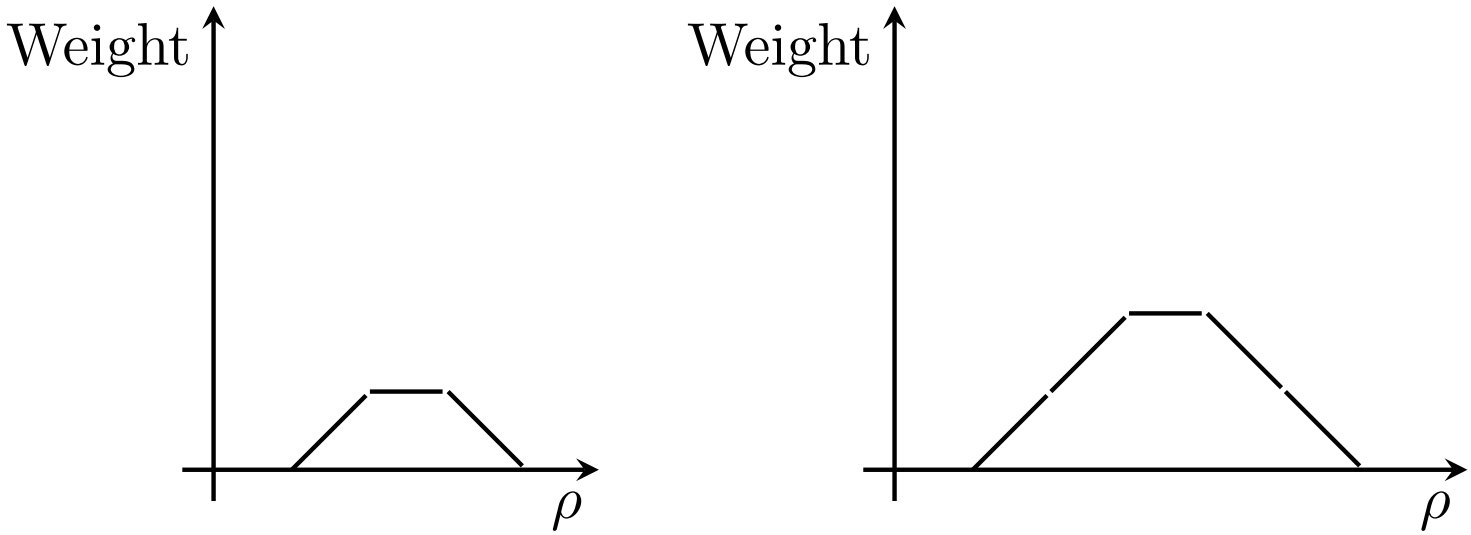
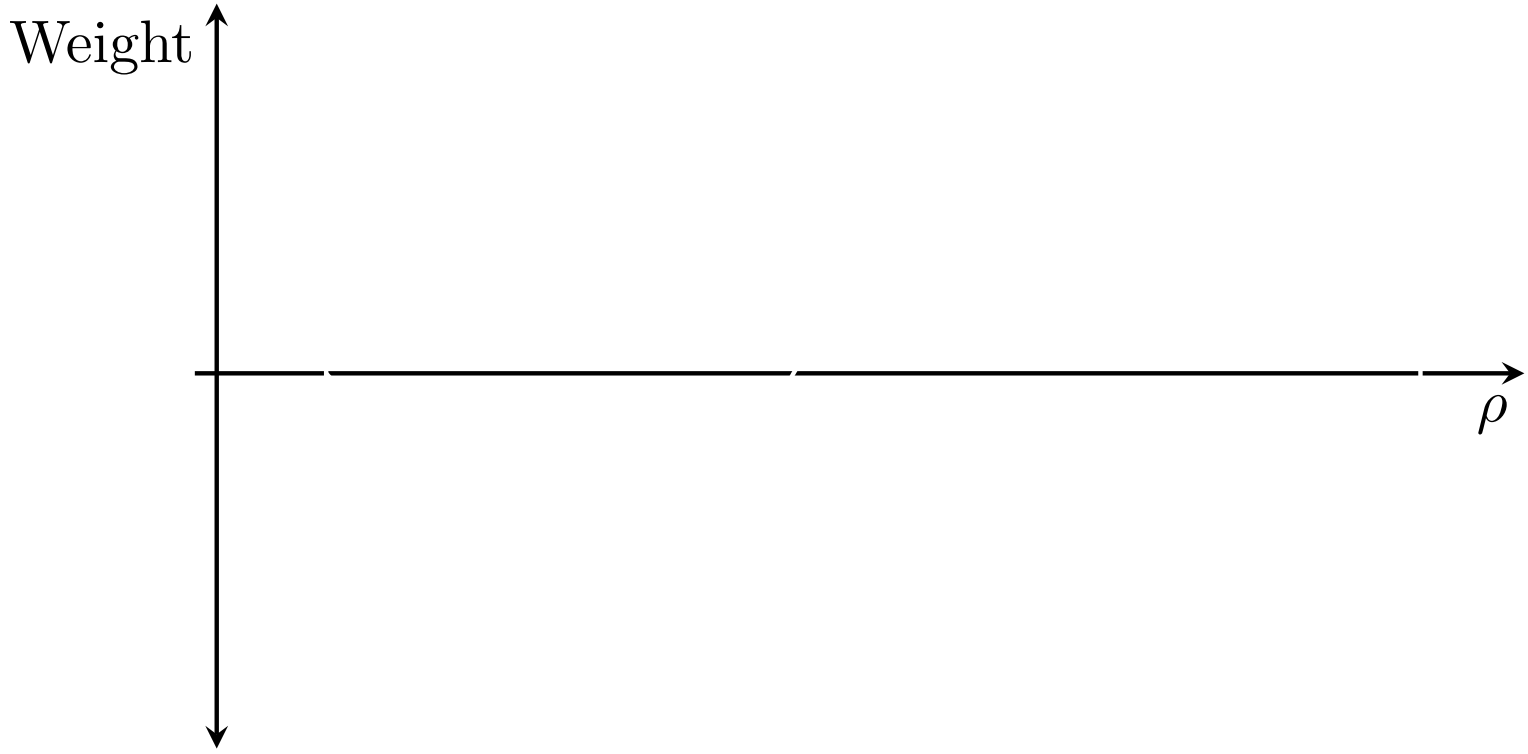




Intuitively:
Measure size of resource required to answer request

Cost Measure



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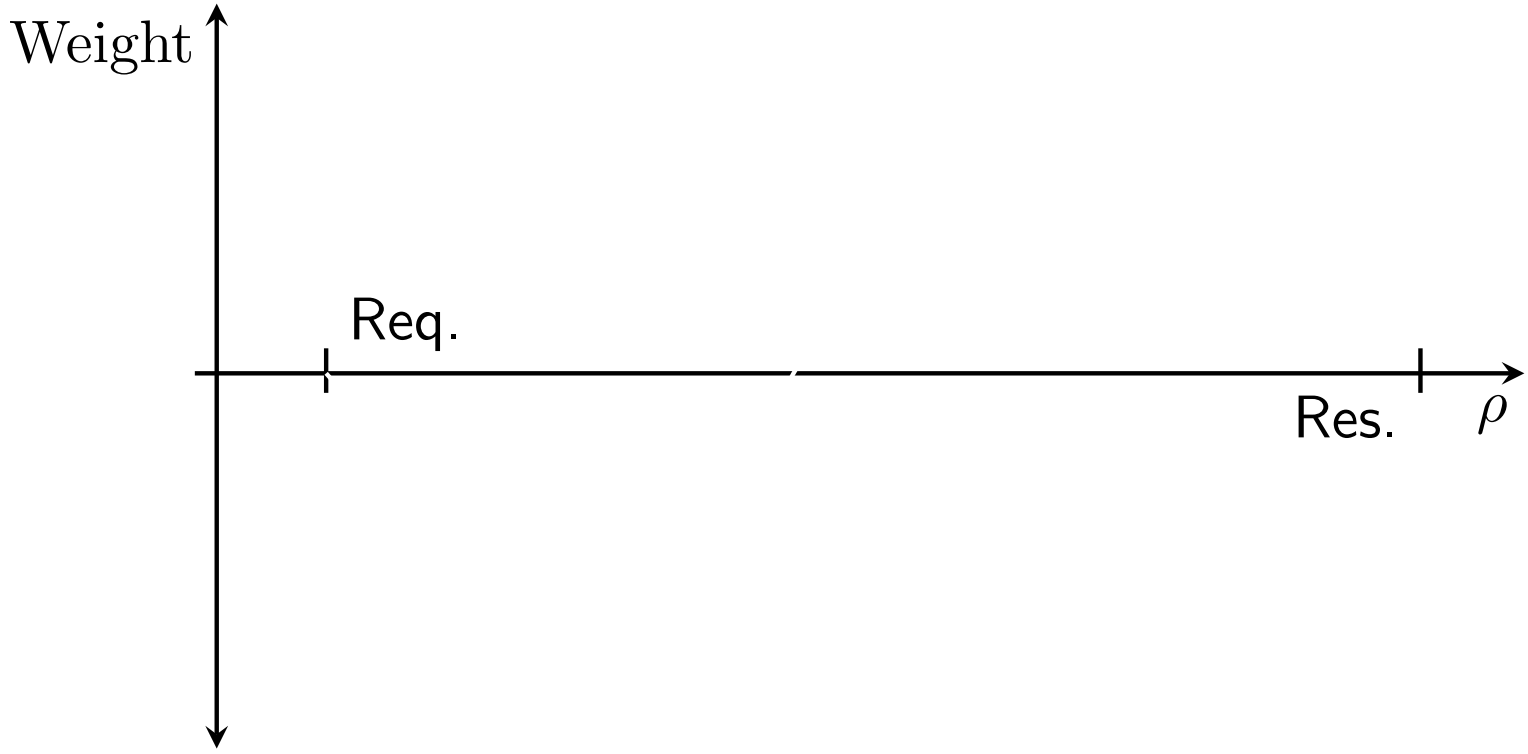

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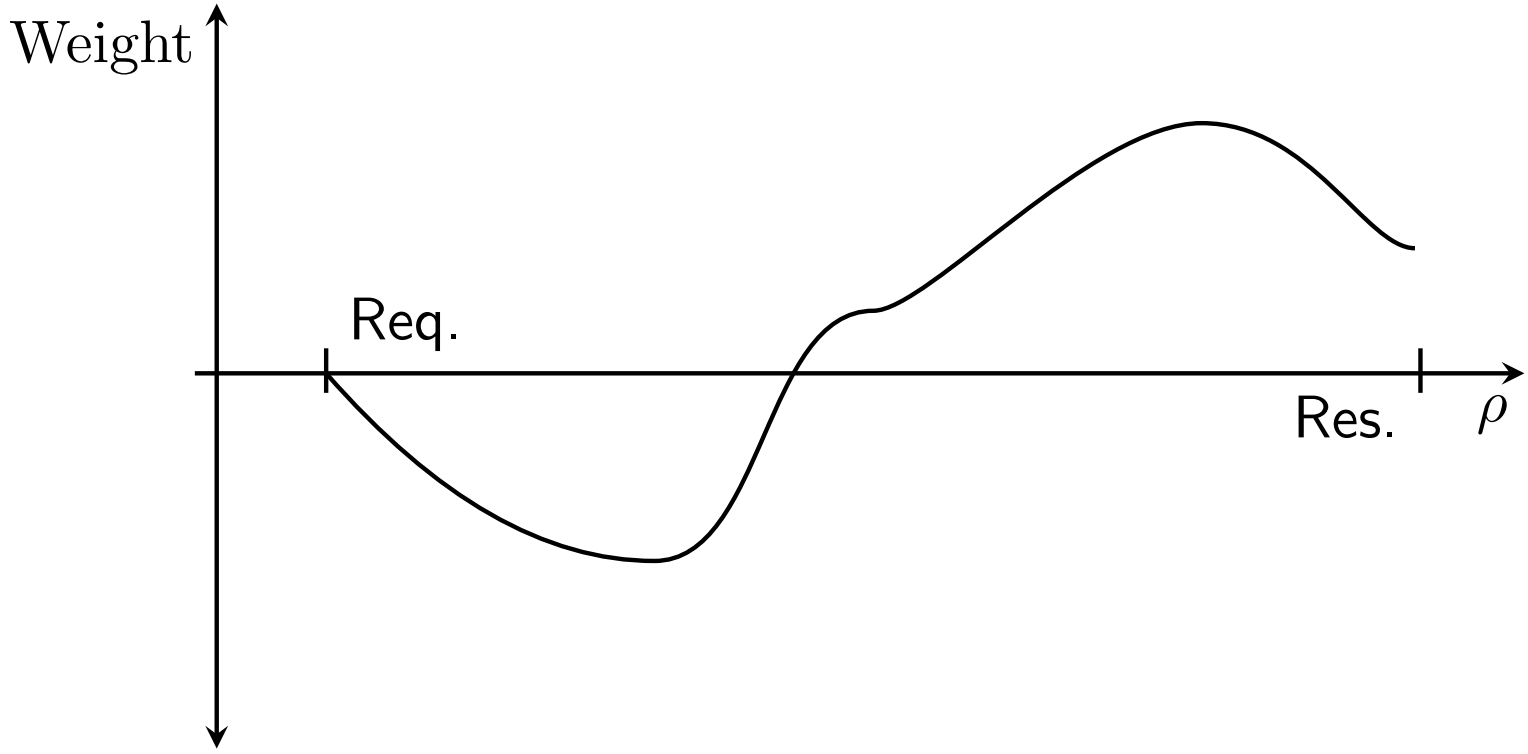

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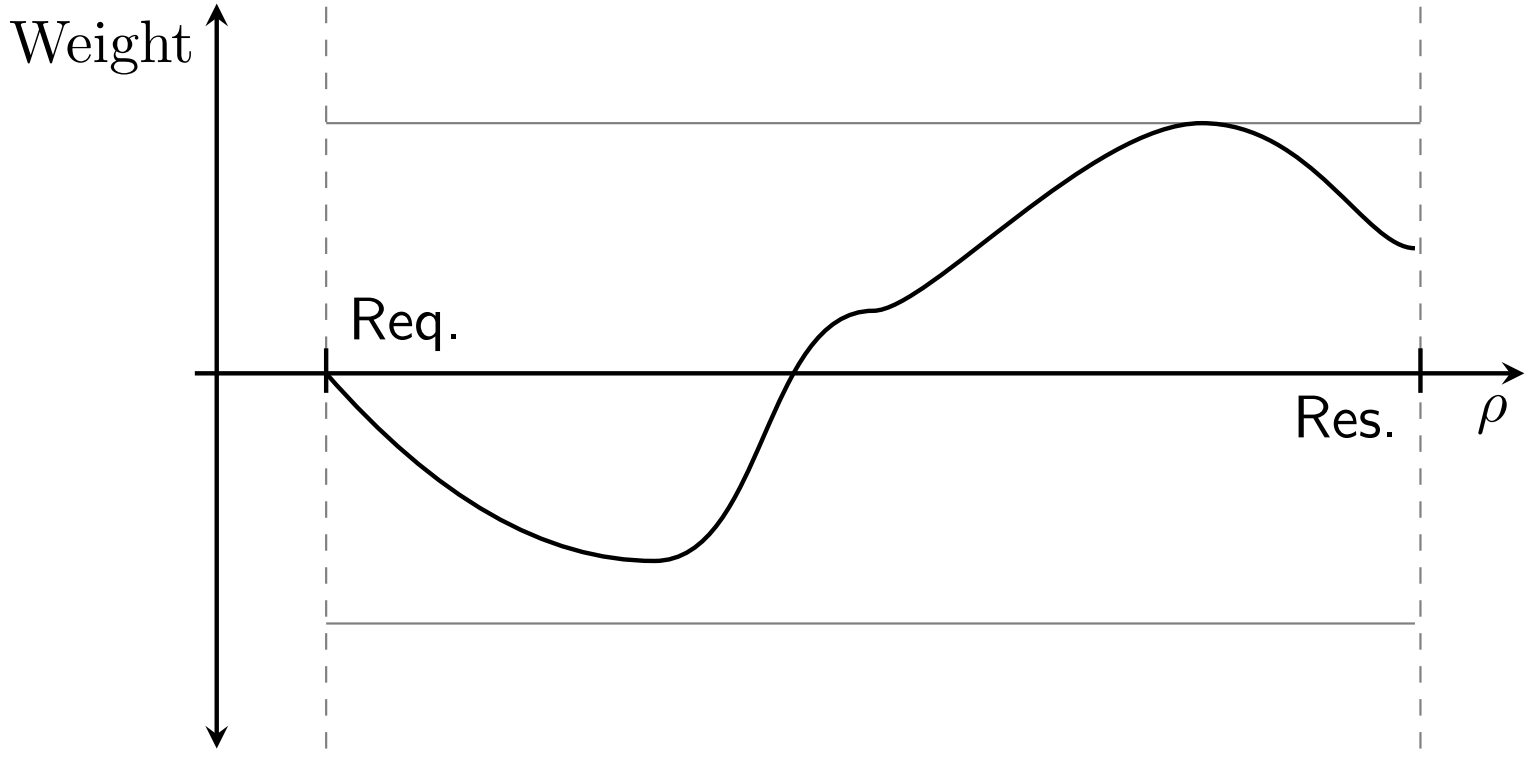

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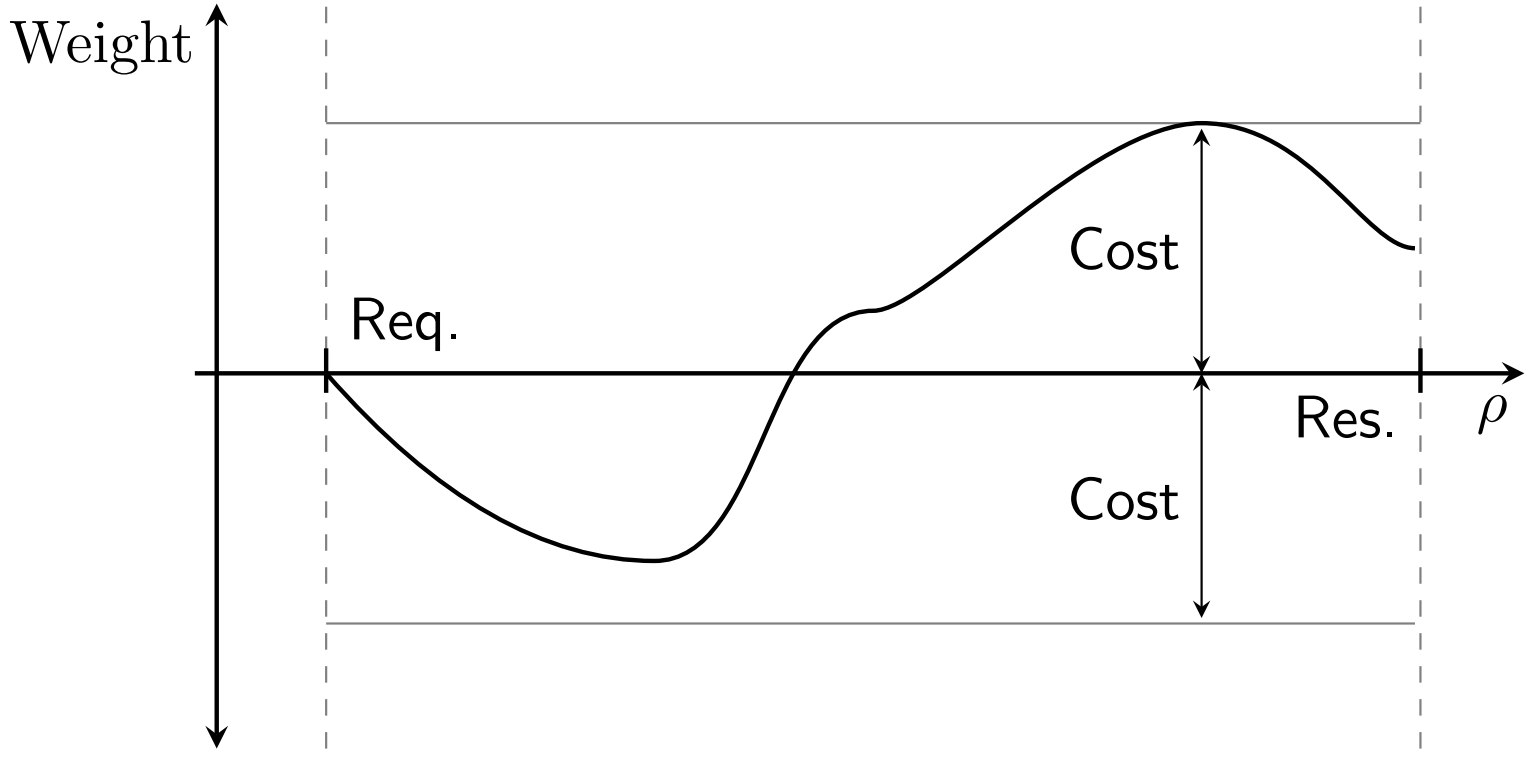

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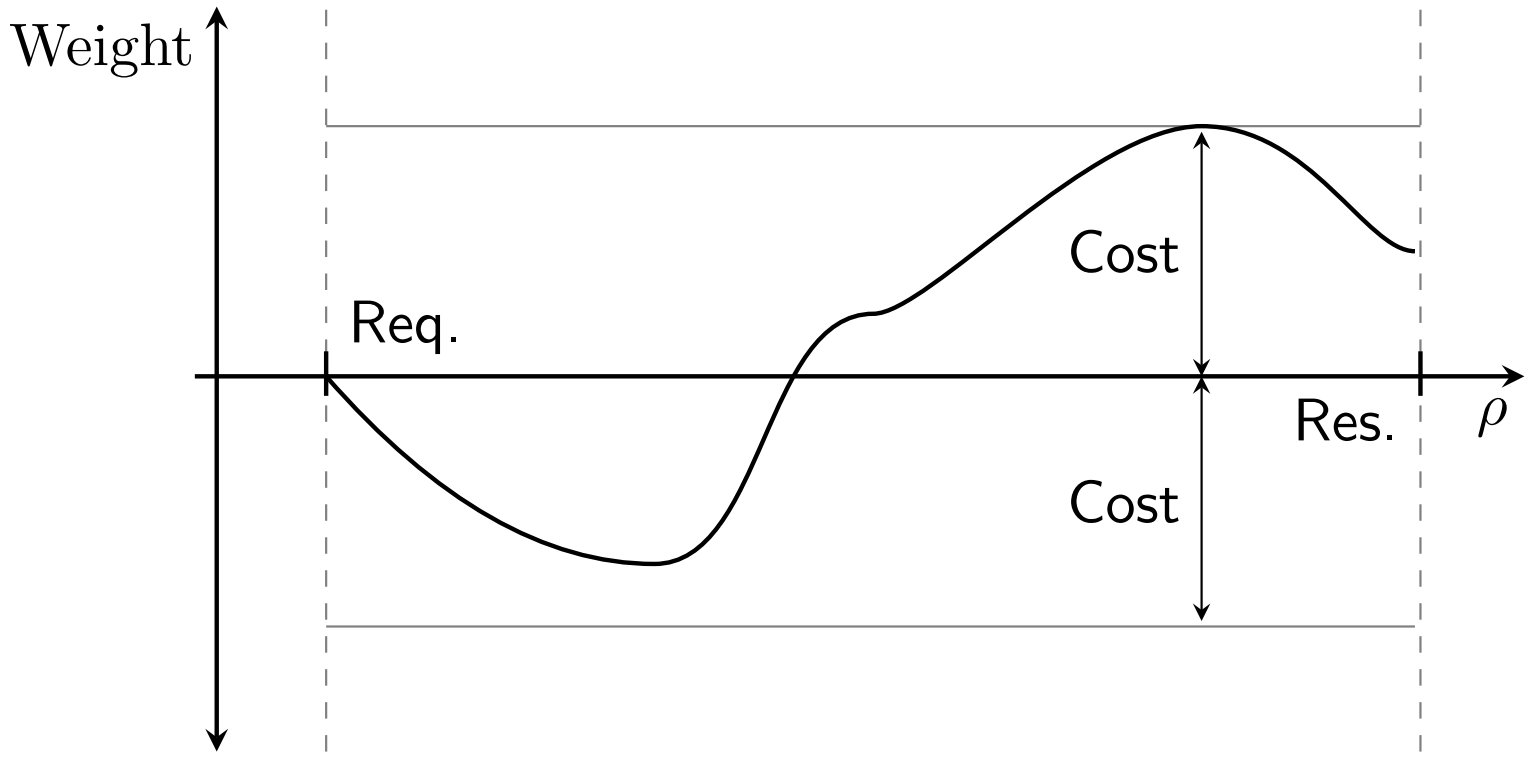

Cost of answering: **Amplitude on path to answer**

Outline

- Definition
- Complexity
- Memory
- Bounds

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Solving Parity Games with Weights

Problem: Given a parity game with weights $\mathcal{G}$ and a vertex v of $\mathcal{G}$, does Player 0 win $\mathcal{G}$ from v?

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Proof idea: Analogously to (Fijalkow and Zimmermann, 2014)

1. Strengthen parity condition with weights
2. Reduce to solving energy parity games

Bounding Parity Games with Weights

First Step: Strengthen parity condition with weights $\rightarrow$ bounded parity games with weights.

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Parity games with weights can be solved by solving polynomially many bounded parity games with weights of polynomial size.

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Solving energy parity games is in $\mathrm{NP} \cap \text{co-}\mathrm{NP}$.

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Solving Bounded Parity Games with Weights

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Energy parity games can be solved in quasi-pseudo-polynomial time.

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Energy parity games can be solved in quasi-pseudo-polynomial time.

Corollary (Daviaud et al., 2018)

Parity games with weights can be solved in quasi-pseudo-polynomial time.

There and Back Again

So far: Solved parity games with weights via iteratively solving energy parity games.

There and Back Again

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Also possible: Solving energy parity games via iteratively solving parity games with weights.

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Proof idea: Again detour via bounded parity games with weights.

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The problems of solving parity games with weights and of solving energy parity games are polynomial-time equivalent.

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Parity Games
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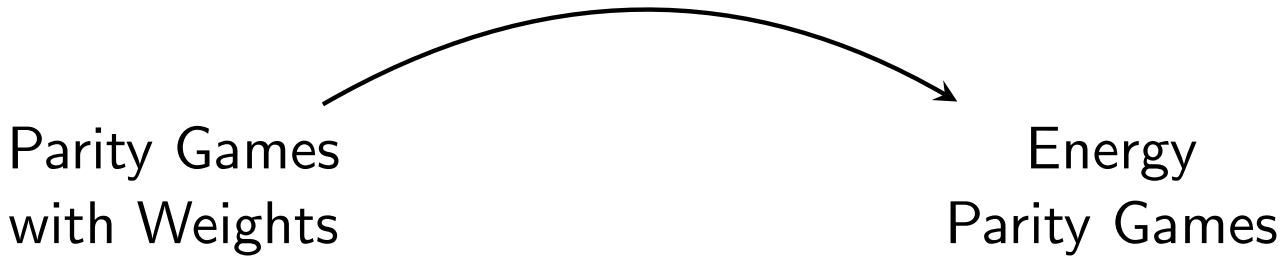

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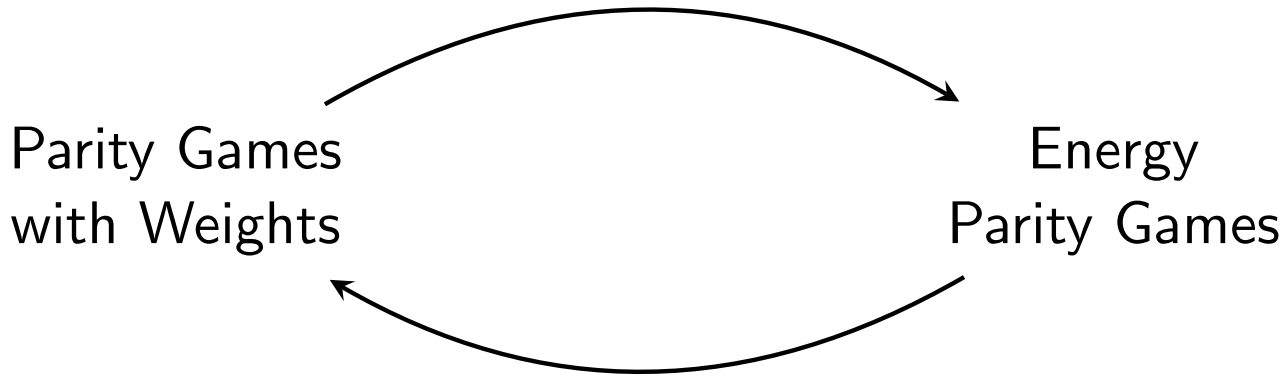

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Intuition:

1. Each energy parity game $\mathcal{G}'_v$ is "tasked" with answering the request posed at vertex v with bounded cost.

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Intuition:

1. Each energy parity game $\mathcal{G}'_v$ is "tasked" with answering the request posed at vertex v with bounded cost.
2. Player 0 wins $\mathcal{G}$ by simulating winning $\mathcal{G}'_v$ for vertex carrying largest requested color.

Memory Requirements (Upper Bound)

Proposition (Chatterjee and Doyen, 2012)

In an energy parity game with n vertices, d odd colors, and largest absolute weight W, memory of size $\mathcal{O}(ndW)$ suffices for her to implement a winning strategy from her winning region.

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In an energy parity game with n vertices, d odd colors, and largest absolute weight W, memory of size $\mathcal{O}(ndW)$ suffices for her to implement a winning strategy from her winning region.

Lemma

In a bounded parity game with weights with n vertices, d odd colors, and largest absolute weight W, memory of size $\mathcal{O}(nd^2W)$ suffices for Player 0 to implement a winning strategy for her from her winning region.

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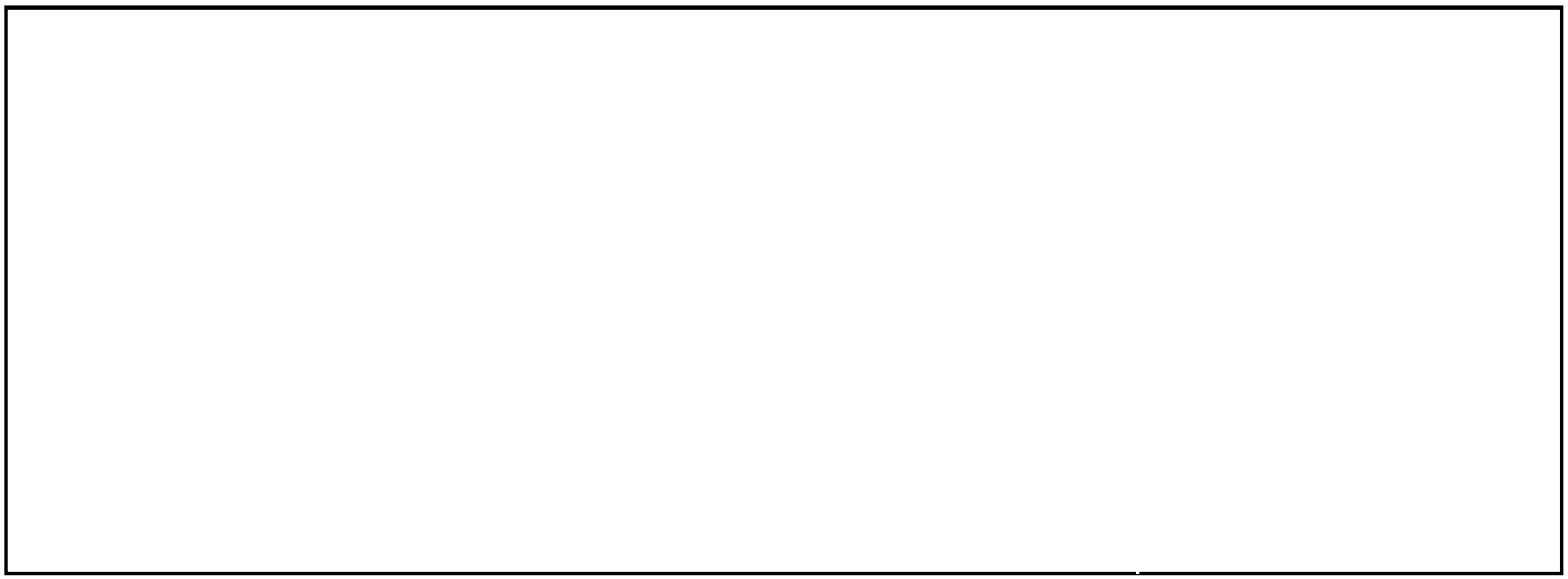

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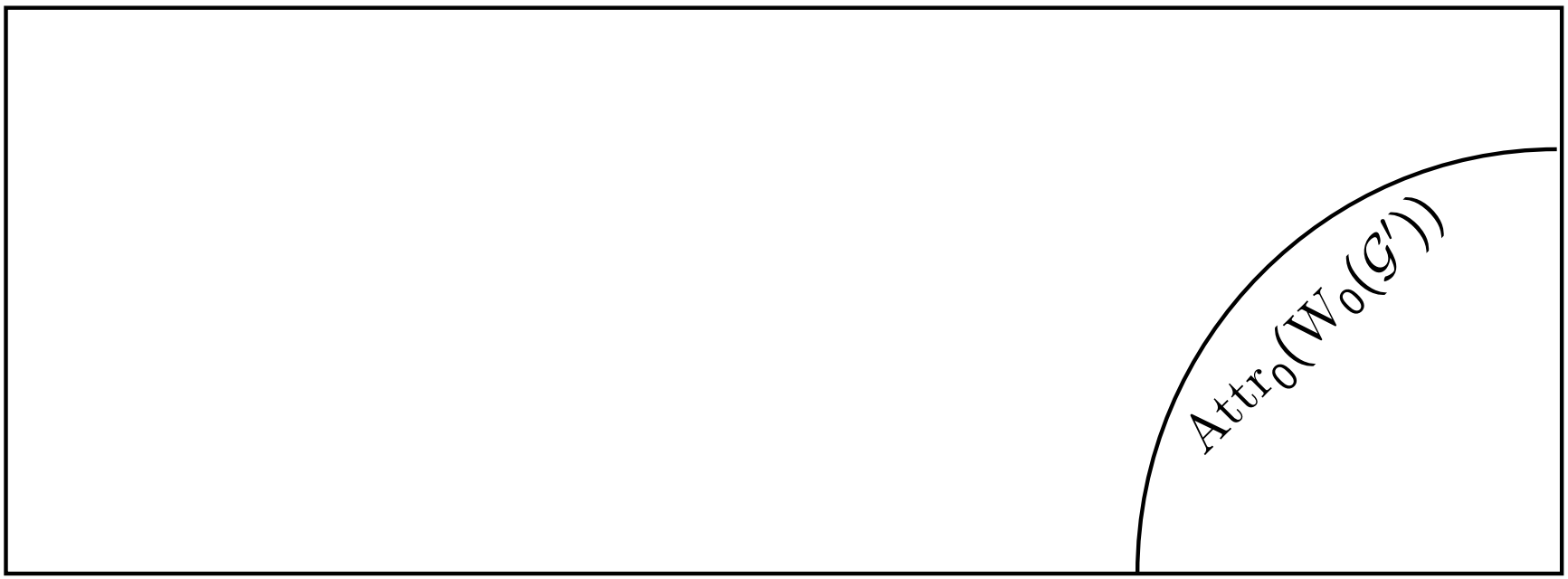

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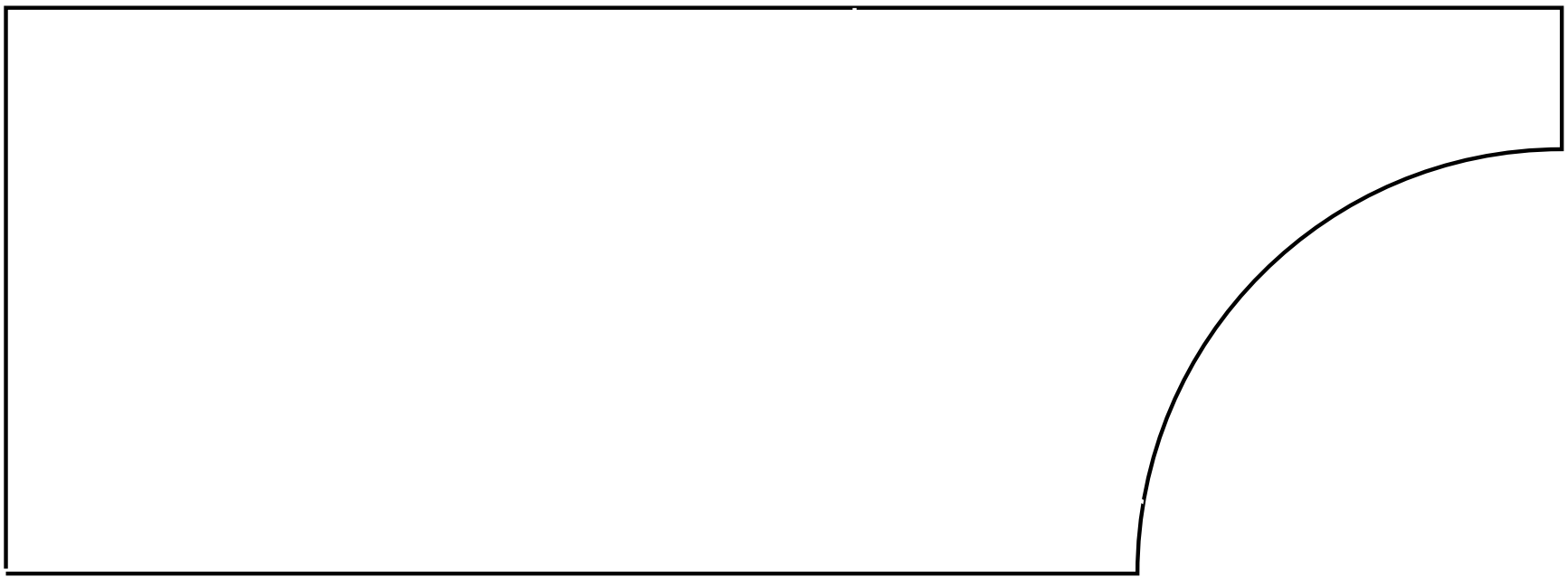

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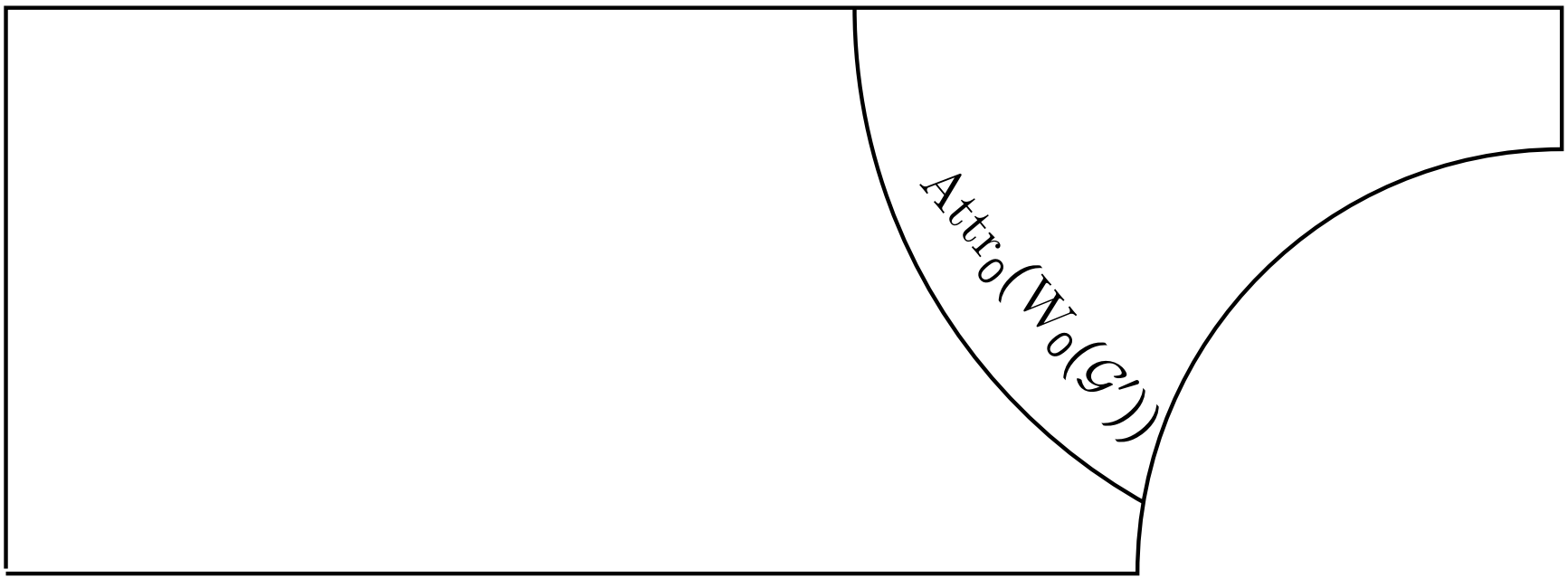

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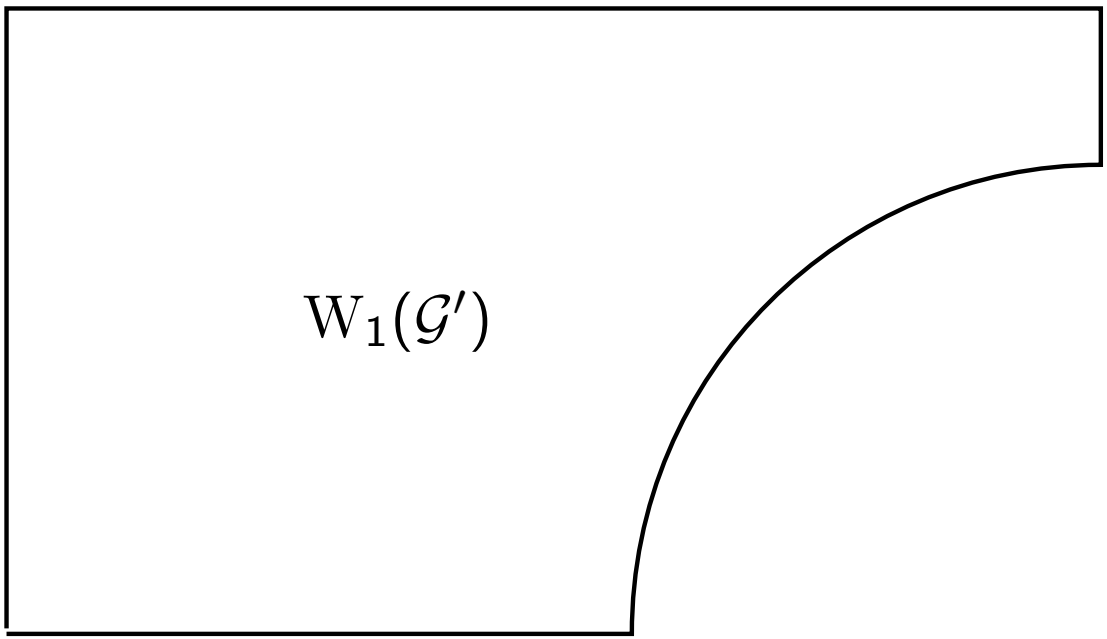

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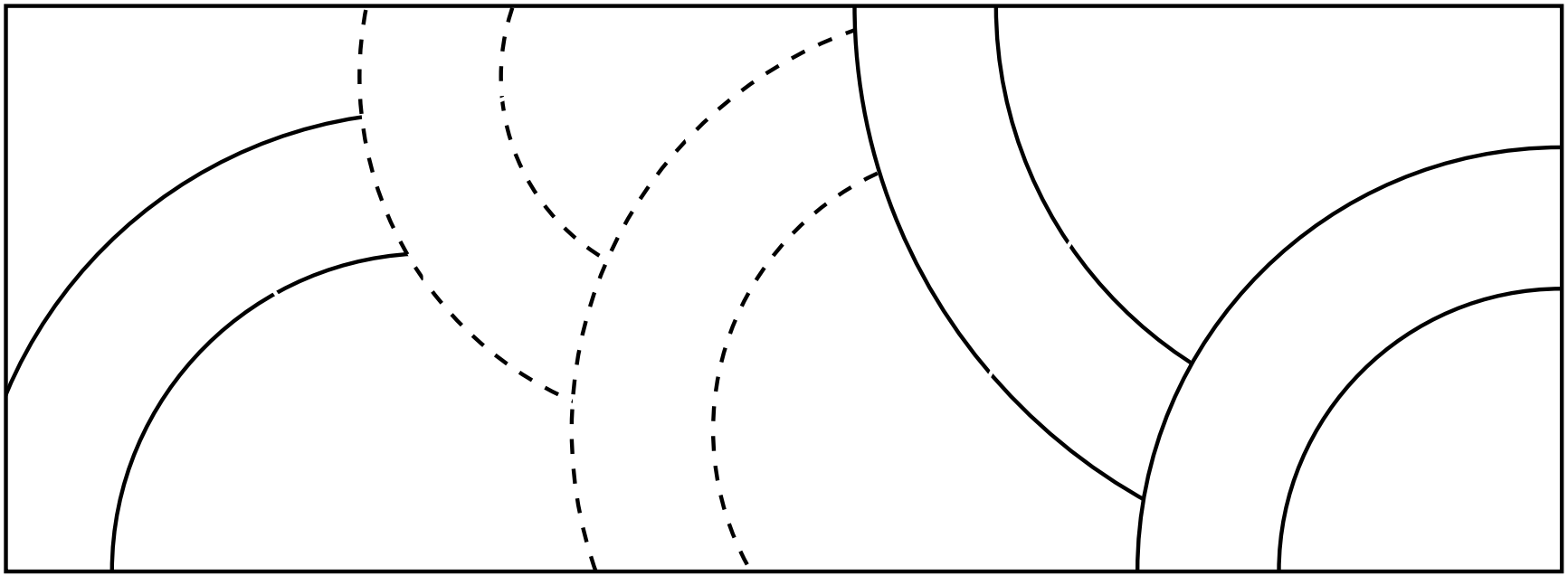

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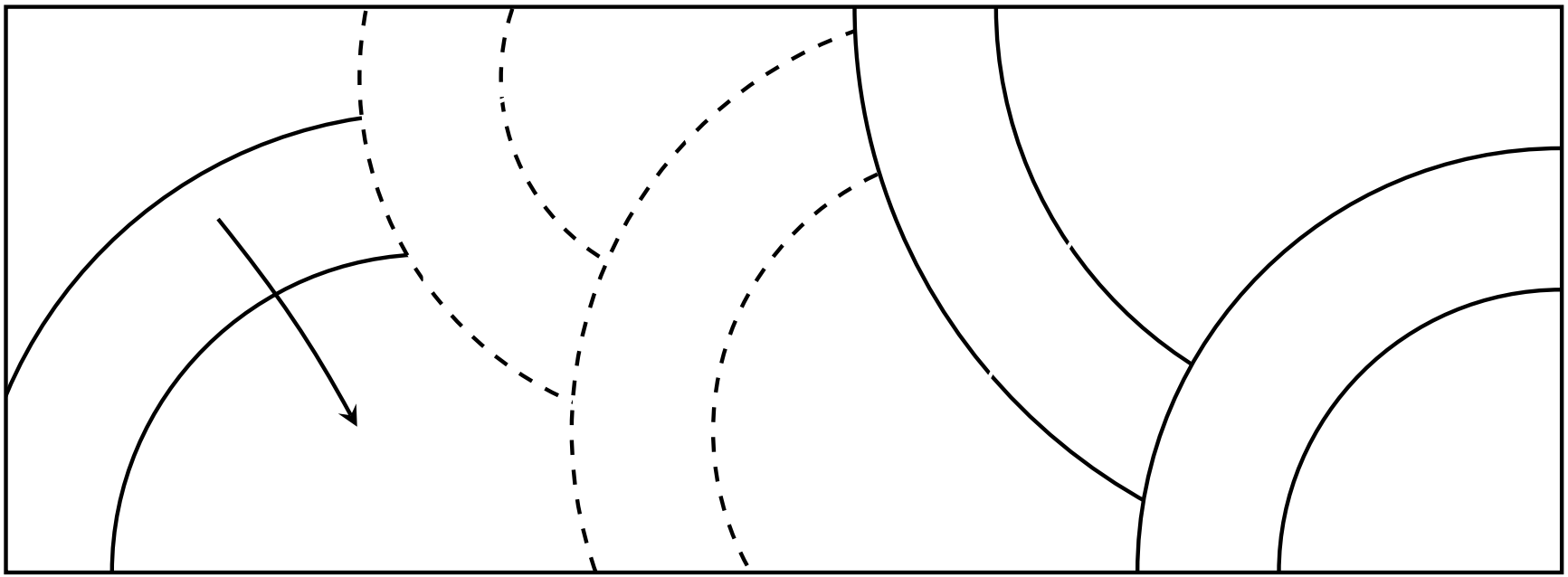

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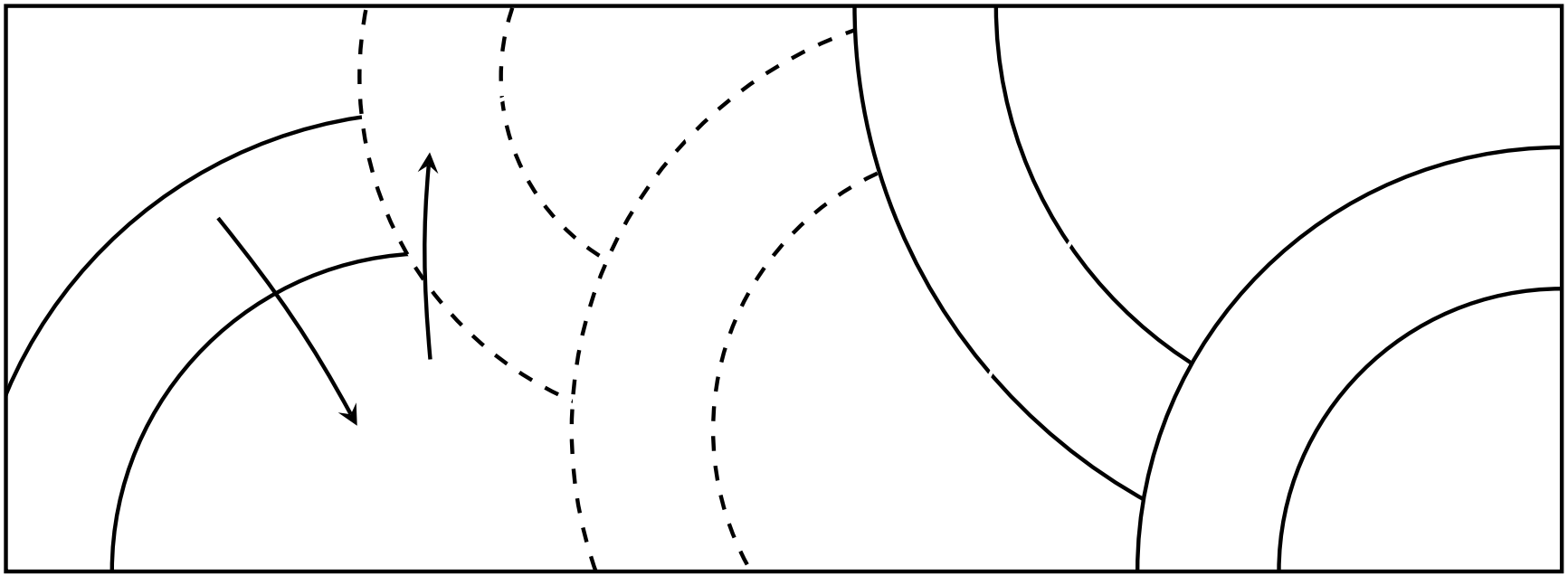

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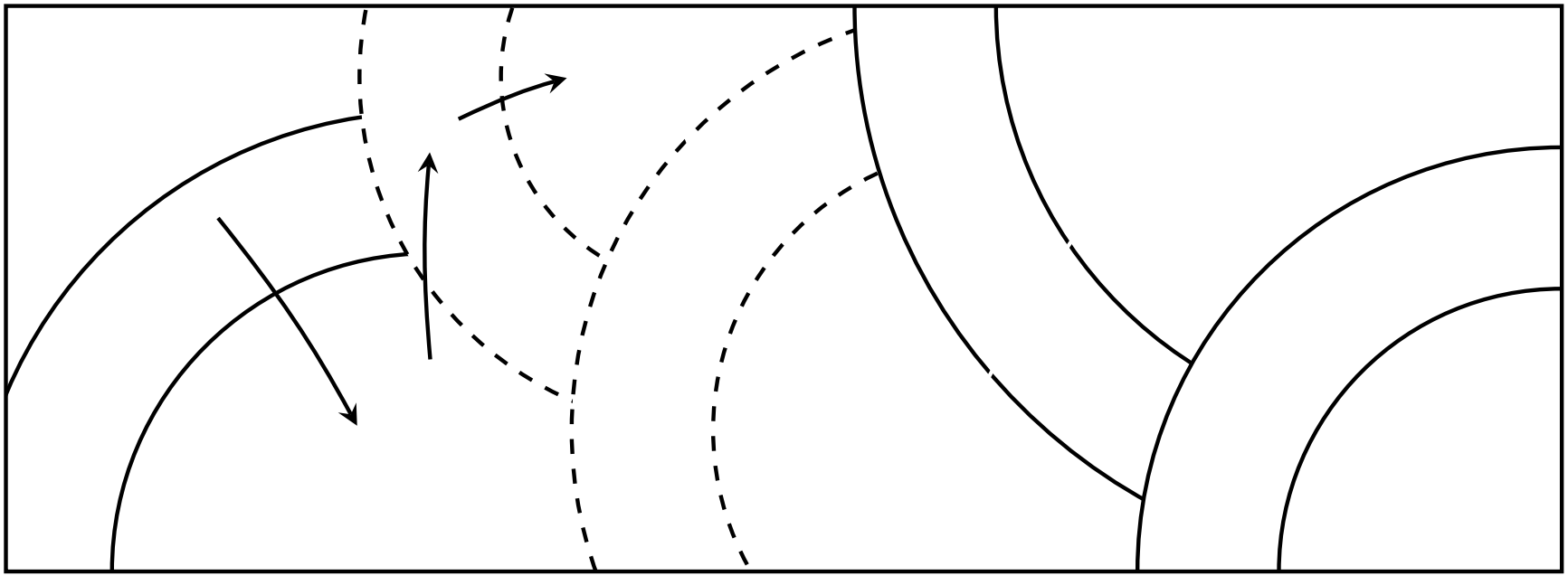

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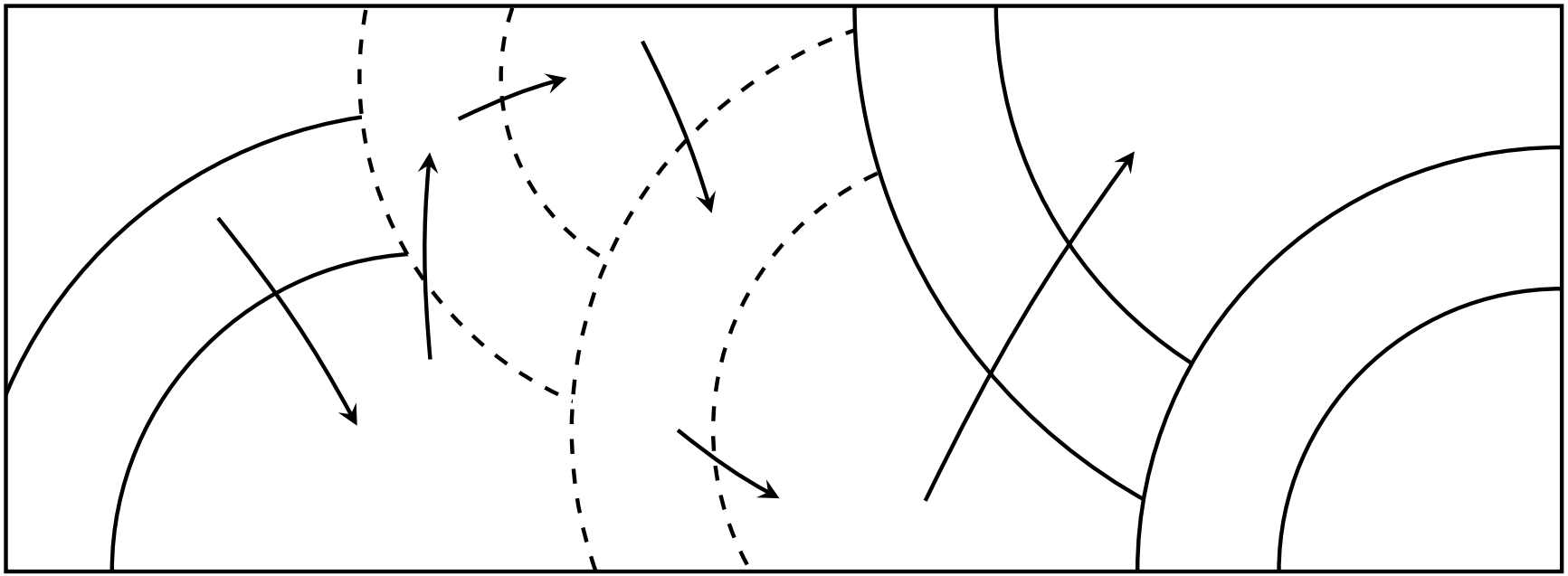
Known so far: Parity games with weights can be solved by solving polynomially many bounded parity games with weights.



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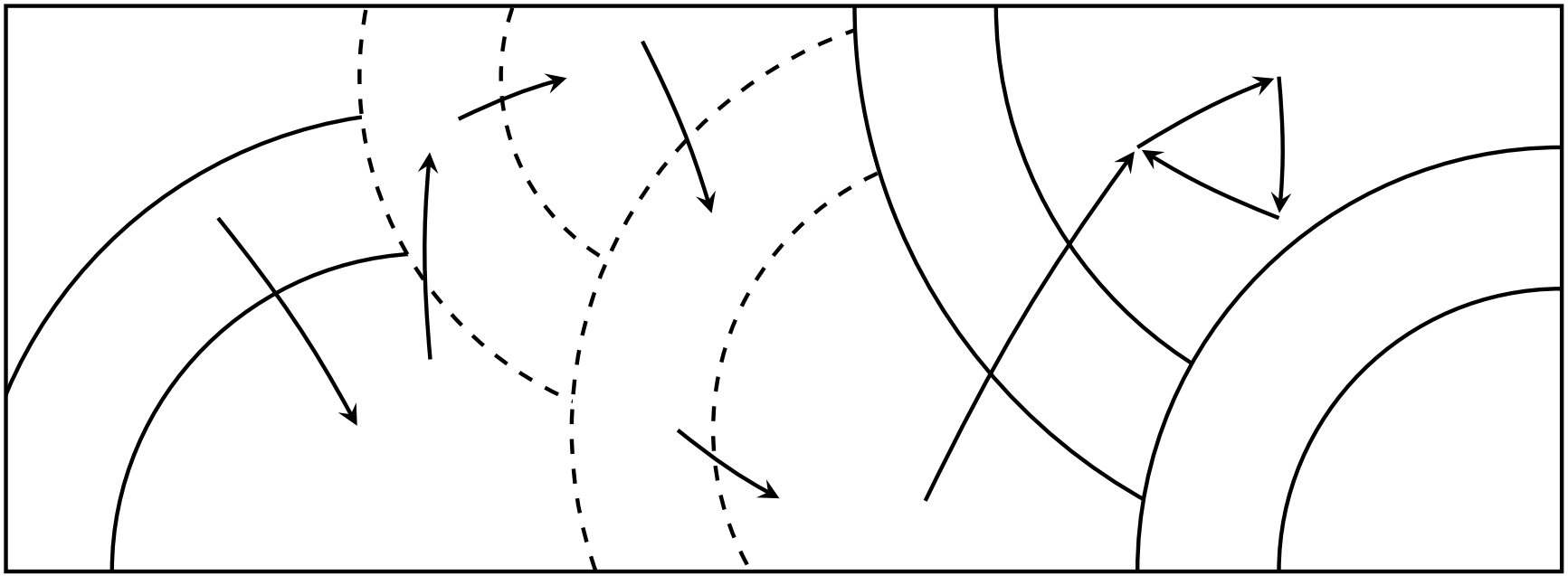

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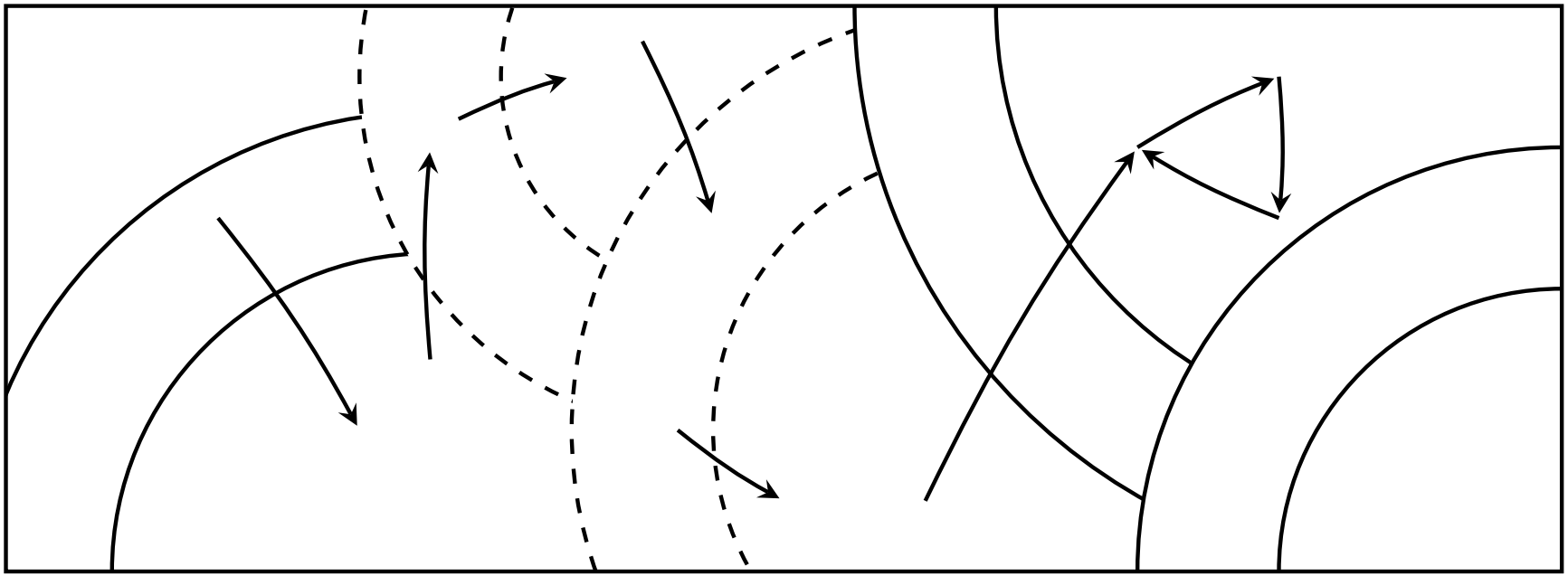

Winning Strategy: At every "border", restart winning bounded parity game with weights

Memory Requirements (Upper Bound)

Lemma

In a bounded parity game with weights with n vertices, d odd colors, and largest absolute weight W, memory of size $\mathcal{O}(nd^2W)$ suffices for Player 0 to implement a winning strategy for her from her winning region.

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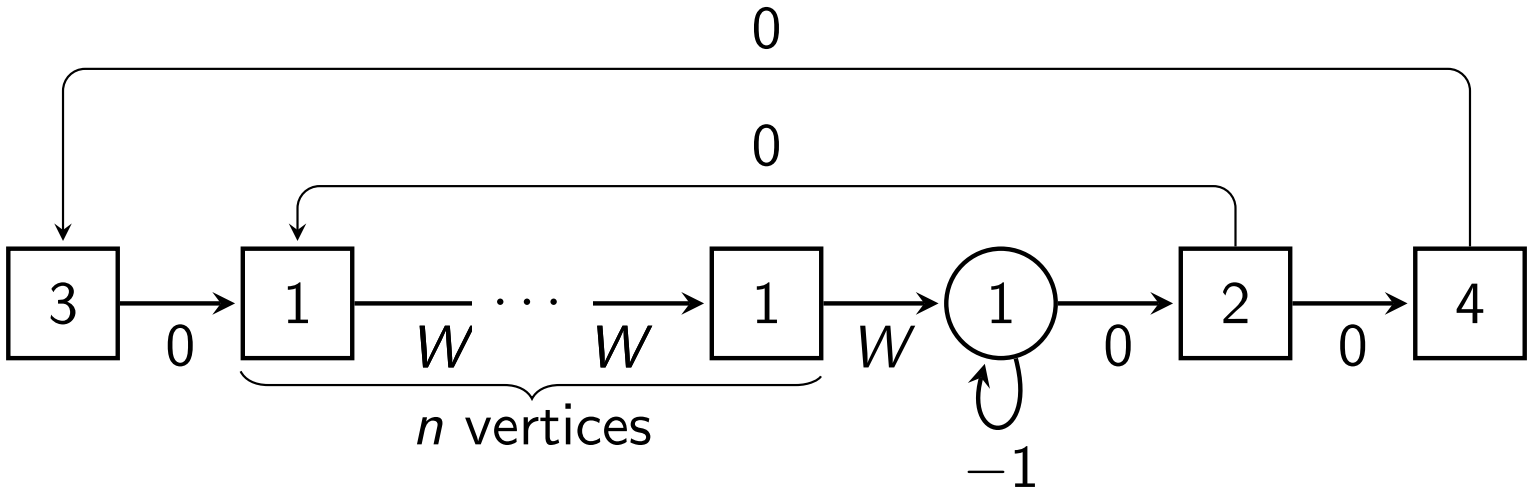
Theorem

In a parity game with weights with n vertices, d odd colors, and largest absolute weight W, memory of size $\mathcal{O}(nd^2W)$ suffices for her to implement a winning strategy from her winning region.

Memory Requirements (Lower Bound)



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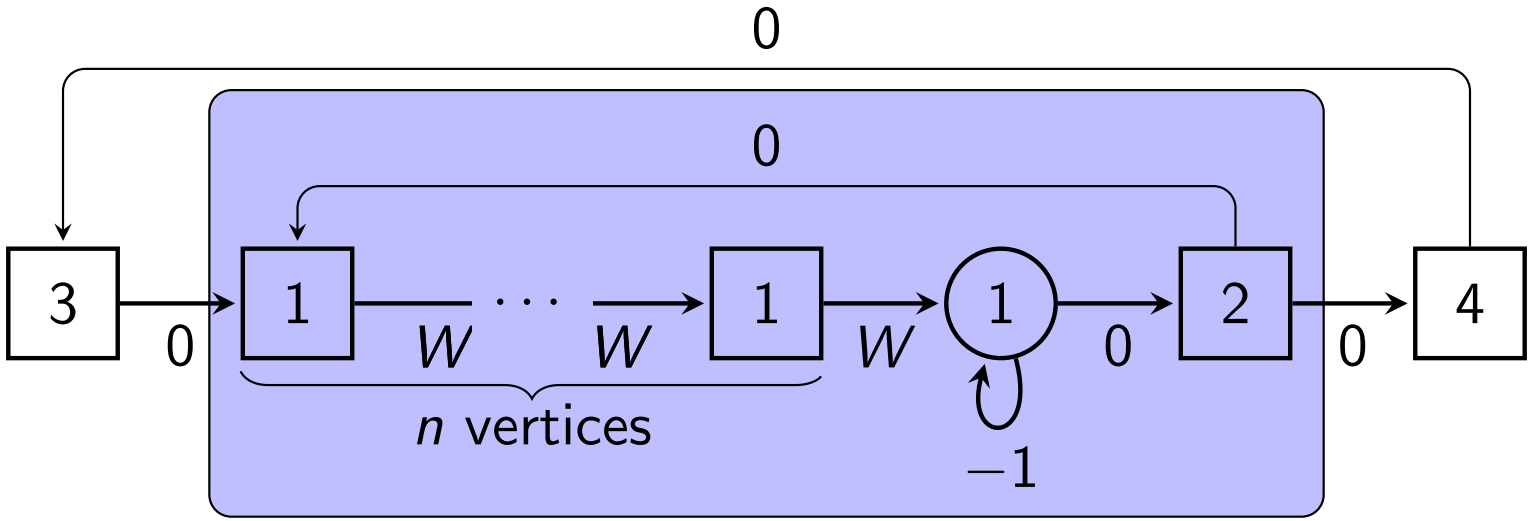

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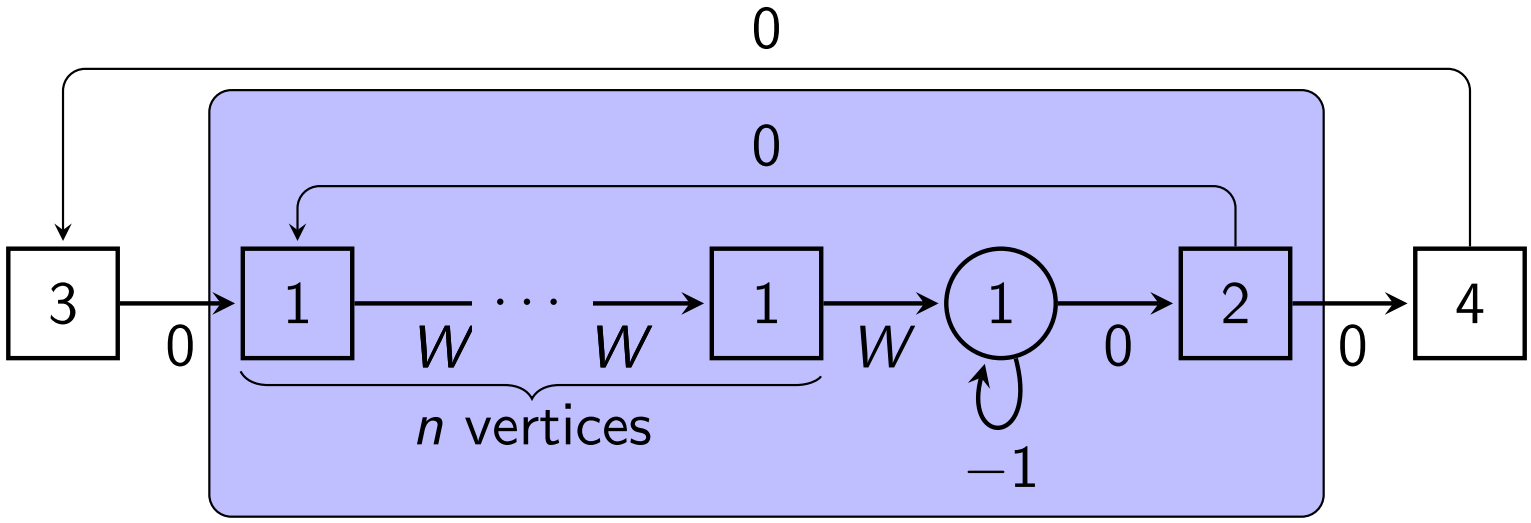

Theorem

Player 0 *requires* $nW + 1$ *memory states to implement a winning strategy in the above game.*

Outline

- Definition ✓
- Complexity ✓
- Memory
- Bounds

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Weight Bounds (Upper Bounds)

Proposition (Chatterjee and Doyen, 2012)

In an energy parity game $\mathcal{G}$ with n vertices, d odd colors, and largest absolute weight W, if Player 0 wins $\mathcal{G}$ from some vertex v, then she has a strategy that bounds the energy level from below by $-nW$.

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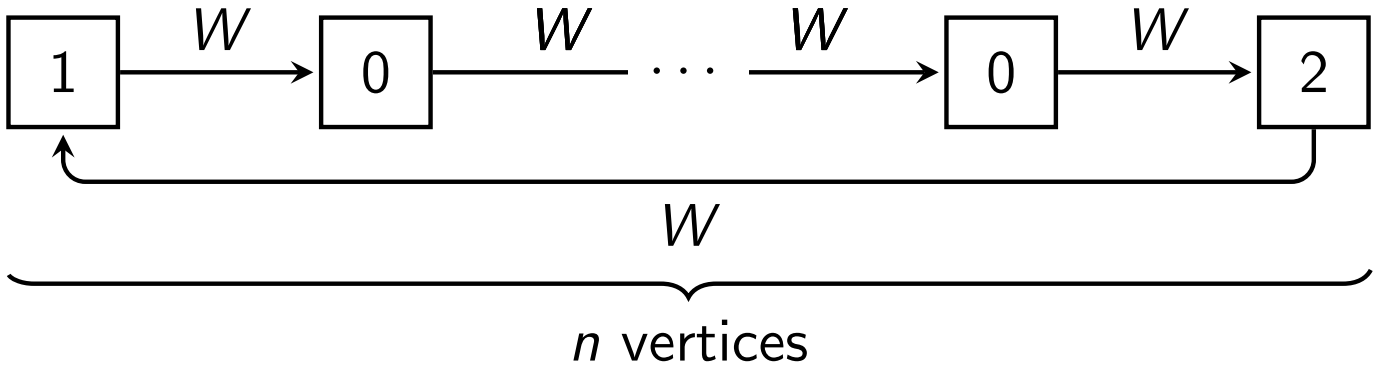
Theorem

In a parity game with weights $\mathcal{G}$ with n vertices, d odd colors, and largest absolute weight W, if Player 0 wins $\mathcal{G}$ from some vertex v, the she has a strategy that bounds the cost of plays by some value in $\mathcal{O}((ndW)^2)$.

Weight Bounds (Lower Bounds)



Theorem

Player 0 *wins the above game, but every play has a cost of* $(n-1)W$.

Conclusion

	Finitary	Costs	Weights
Complexity	PTIME	$\mathrm{UP} \cap \mathrm{co\text{-}UP}$	

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Future Work:

- Tighten bounds
  - Lower bound on memory: $nW + 1$
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