Parity Games with Weights

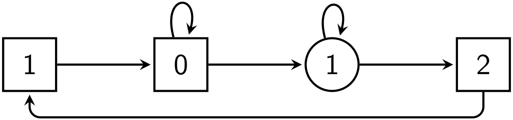
Joint work with Sven Schewe (University of Liverpool)
and Martin Zimmermann (Saarland University)

Alexander Weinert

Saarland University

September 19th, 2018

Parity Games

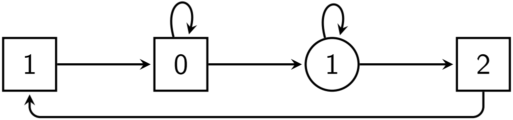


$$\underbrace{1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow}_{\text{Max. color inftly often even}} 1 \longrightarrow 1 \longrightarrow 2 \rightarrow \cdots$$

Proposition (Jurdziński 1998, Calude et al. 2017)

Solving parity games is in $\mathrm{UP} \cap \textsc{co}\text{-}\mathrm{UP}$ *and they can be solved in quasi-polynomial time. Both players have memoryless winning strategies.*

Finitary Parity Games



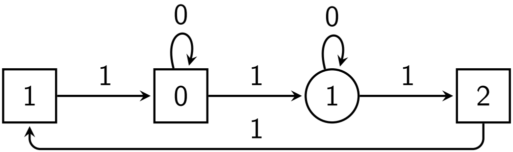
$$\underbrace{1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2}_{\text{3 steps}} \longrightarrow \underbrace{1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 1 \longrightarrow 1 \longrightarrow 2}_{\text{5 steps}} \rightarrow \cdots$$

Aim for Player 0:
Eventually bound steps between requests and answers

Proposition (Chatterjee, Henzinger, and Horn, 2009)

Solving finitary parity games is in PTIME. *Player* 0 *has memoryless winning strategies.*

Parity Games with Costs



$$\underbrace{1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2}_{\text{Weight } 3} \longrightarrow \underbrace{1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 1 \longrightarrow 1 \longrightarrow 2}_{\text{Weight } 3} \longrightarrow \cdots$$

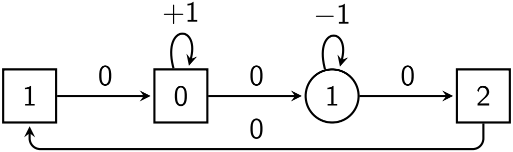
Aim for Player 0:

Eventually bound ~~steps~~ weight between request and answer

Proposition (Fijalkow and Zimmermann 2014 / Mogavero, Murano, and Sorrentino 2015)

Solving parity games with costs is in $\mathrm{UP} \cap \mathrm{co}\text{-}\mathrm{UP}$. *Player* 0 *has memoryless winning strategies.*

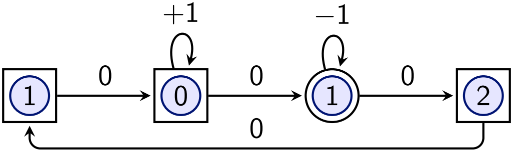
Parity Games with Weights

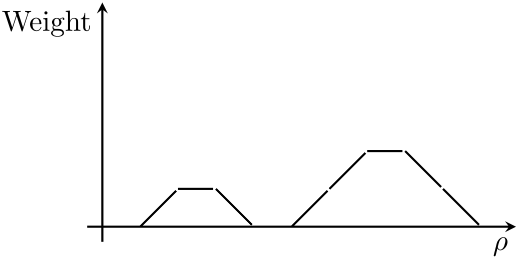


$$\underbrace{1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 2}_{\text{Weight } 0} \longrightarrow \underbrace{1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 1 \longrightarrow 1 \longrightarrow 2}_{\text{Weight } 0} \longrightarrow \cdots$$

Not the complete picture...

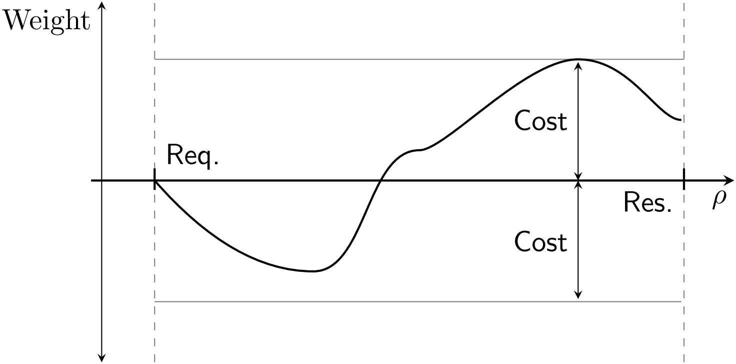
Cost Measure





Intuitively:
Measure size of resource required to answer request

Cost Measure



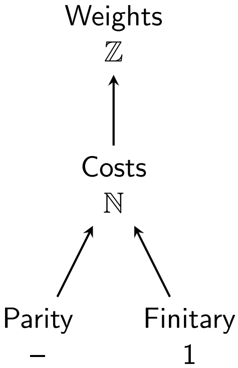
Cost of answering: **Amplitude on path to answer**

Goal for Player 0: Bound cost of almost all requests

Results

	Complexity	Memory	Bounds
Finitary	PTIME	positional	n
Costs	$\mathrm{UP} \cap \textsc{co}\text{-}\mathrm{UP}$	positional	nW
Weights	$\mathrm{NP} \cap \textsc{co}\text{-}\mathrm{NP}$	$\mathcal{O}(nd^2W)$	$\mathcal{O}((ndW)^2)$

Conclusion



Weights
$\mathbb{Z}$

Costs
$\mathbb{N}$

Parity
–

Finitary
1

Take-Away: Greater expressiveness incurs

- small cost in terms of solving
- great cost in size of strategy