

Parity Games with Weights

Joint work with Sven Schewe (University of Liverpool)
and Martin Zimmermann (Saarland University)

Alexander Weinert

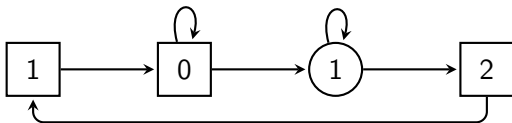
Saarland University

September 19th, 2018

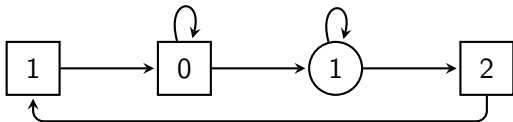
Parity Games



Parity Games

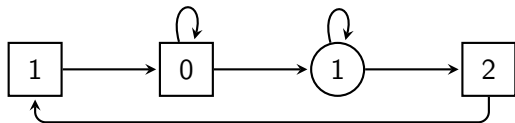


Parity Games



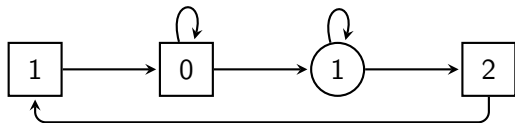
1 → 0 → 1 → 2 → 1 → 0 → 0 → 1 → 1 → 2 → ...

Parity Games



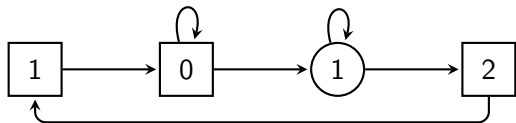
$1 \rightarrow 0 \rightarrow 1 \rightarrow 2 \rightarrow 1 \rightarrow 0 \rightarrow 0 \rightarrow 1 \rightarrow 1 \rightarrow 2 \rightarrow \dots$

Parity Games



$1 \rightarrow 0 \rightarrow 1 \rightarrow 2 \rightarrow 1 \rightarrow 0 \rightarrow 0 \rightarrow 1 \rightarrow 1 \rightarrow 2 \rightarrow \dots$
Max. color infly often even

Parity Games

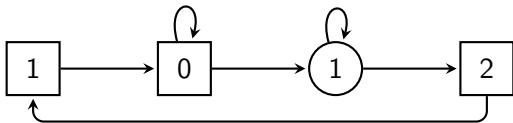


$1 \rightarrow 0 \rightarrow 1 \rightarrow 2 \rightarrow 1 \rightarrow 0 \rightarrow 0 \rightarrow 1 \rightarrow 1 \rightarrow 2 \rightarrow \dots$
Max. color infty often even

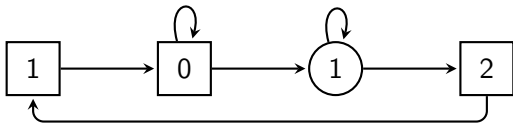
Proposition (Jurdziński 1998, Calude et al. 2017)

Solving parity games is in $UP \cap co-UP$ and they can be solved in quasi-polynomial time. Both players have memoryless winning strategies.

Finitary Parity Games

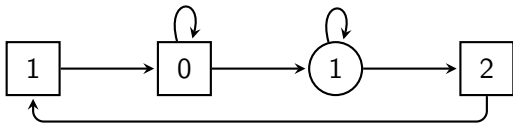


Finitary Parity Games



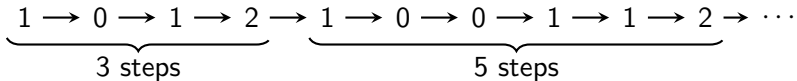
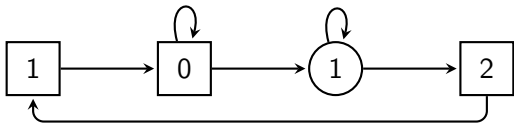
$1 \rightarrow 0 \rightarrow 1 \rightarrow 2 \rightarrow 1 \rightarrow 0 \rightarrow 0 \rightarrow 1 \rightarrow 1 \rightarrow 2 \rightarrow \dots$

Finitary Parity Games

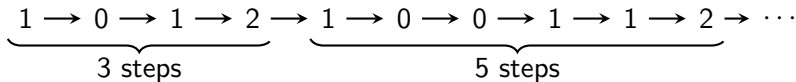
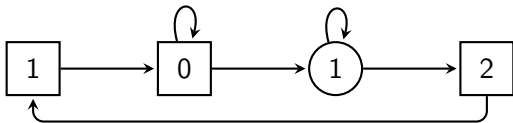


$1 \rightarrow 0 \rightarrow 1 \rightarrow 2 \rightarrow 1 \rightarrow 0 \rightarrow 0 \rightarrow 1 \rightarrow 1 \rightarrow 2 \rightarrow \dots$
3 steps

Finitary Parity Games

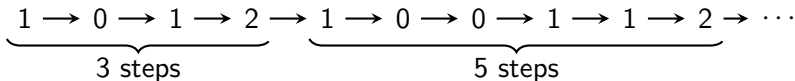
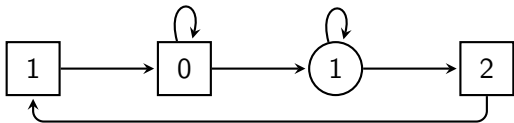


Finitary Parity Games



Aim for Player 0:
Eventually bound steps between requests and answers

Finitary Parity Games

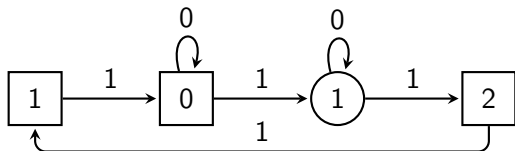


Aim for Player 0:
Eventually bound steps between requests and answers

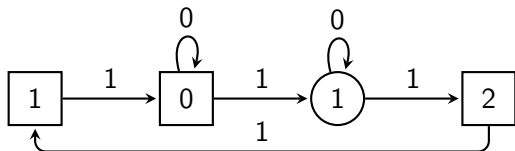
Proposition (Chatterjee, Henzinger, and Horn, 2009)

Solving finitary parity games is in PTIME. Player 0 has memoryless winning strategies.

Parity Games with Costs

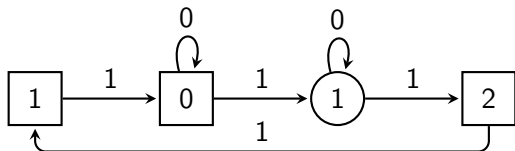


Parity Games with Costs



1 → 0 → 1 → 2 → 1 → 0 → 0 → 1 → 1 → 2 → ...

Parity Games with Costs

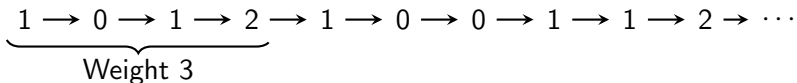
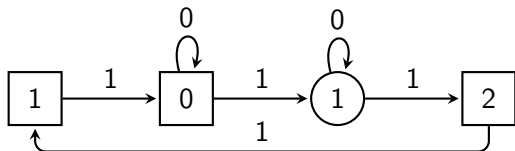


1 → 0 → 1 → 2 → 1 → 0 → 0 → 1 → 1 → 2 → ...

Aim for Player 0:

Eventually bound steps between request and answer

Parity Games with Costs

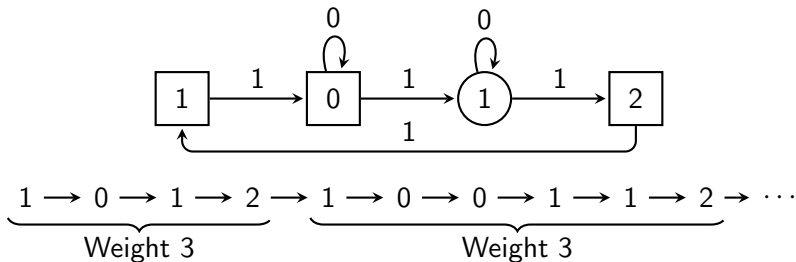


Aim for Player 0:

Eventually bound steps

between request and answer

Parity Games with Costs

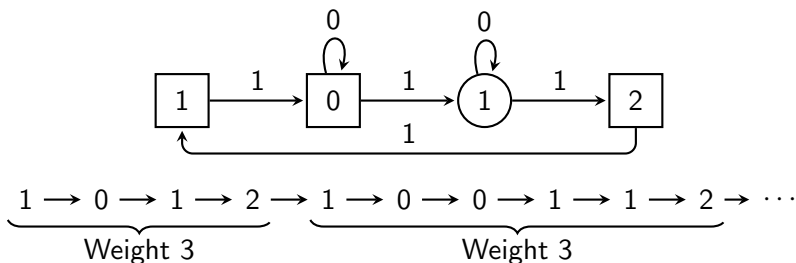


Aim for Player 0:

Eventually bound steps

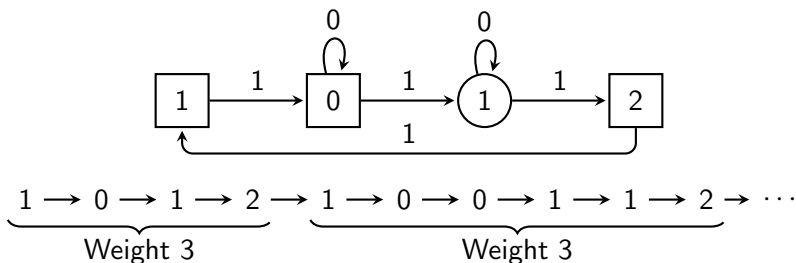
between request and answer

Parity Games with Costs



Aim for Player 0:
Eventually bound steps **weight** between request and answer

Parity Games with Costs



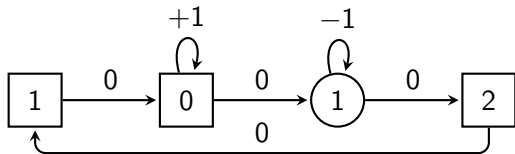
Aim for Player 0:

Eventually bound steps **weight** between request and answer

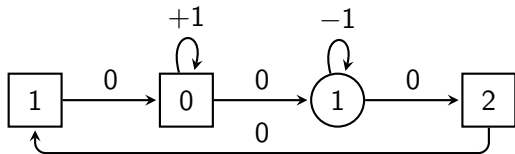
**Proposition (Fijalkow and Zimmermann 2014 /
Mogavero, Murano, and Sorrentino 2015)**

Solving parity games with costs is in $UP \cap co-UP$. Player 0 has memoryless winning strategies.

Parity Games with Weights

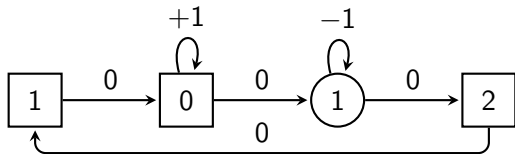


Parity Games with Weights



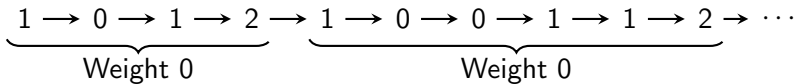
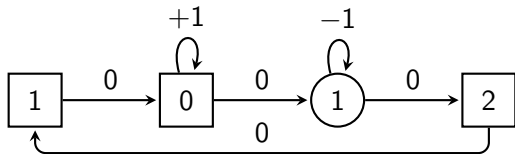
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Parity Games with Weights

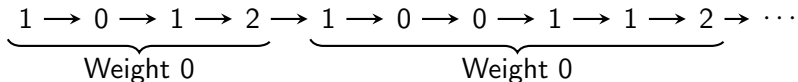
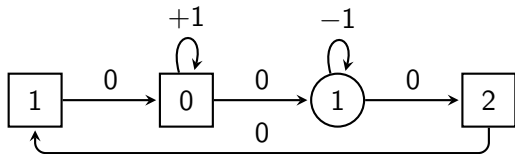


$1 \rightarrow 0 \rightarrow 1 \rightarrow 2 \rightarrow 1 \rightarrow 0 \rightarrow 0 \rightarrow 1 \rightarrow 1 \rightarrow 2 \rightarrow \dots$
Weight 0

Parity Games with Weights

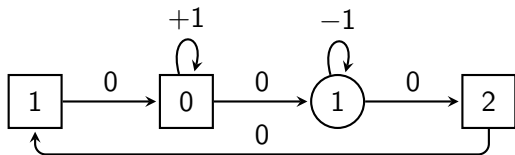


Parity Games with Weights

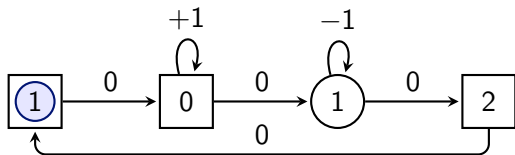


Not the complete picture...

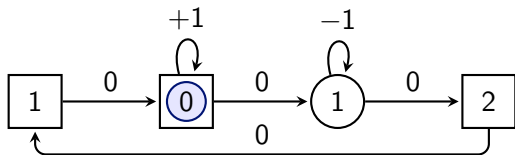
Cost Measure



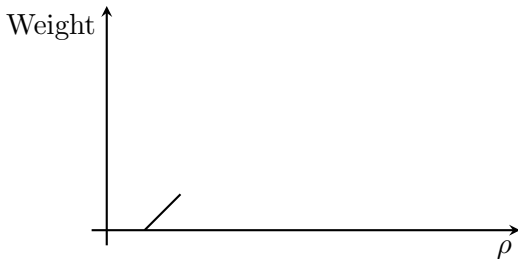
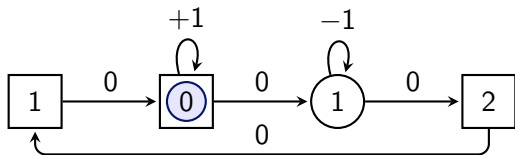
Cost Measure



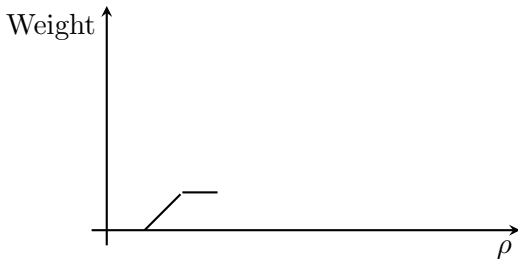
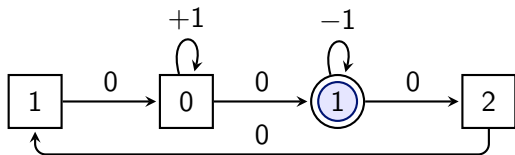
Cost Measure



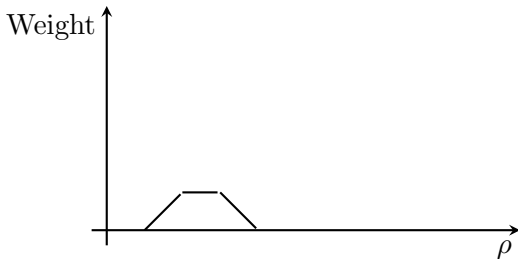
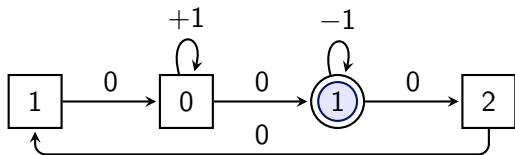
Cost Measure



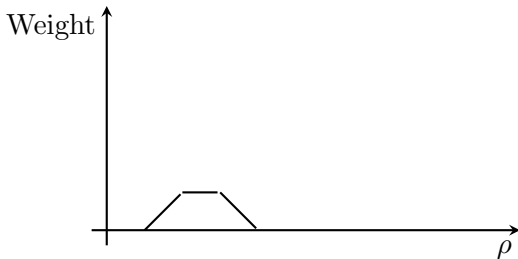
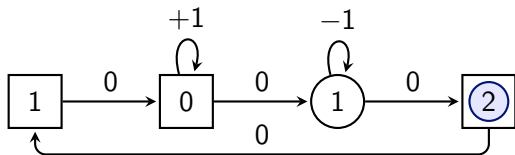
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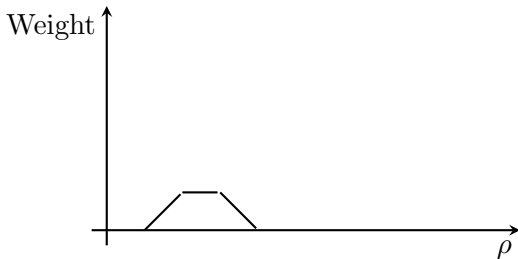
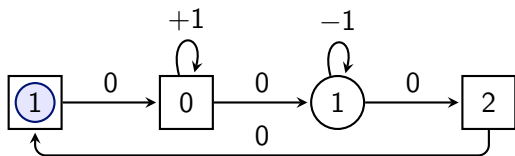
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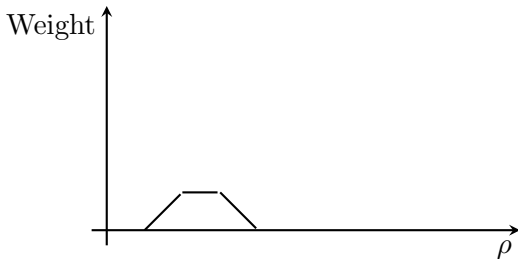
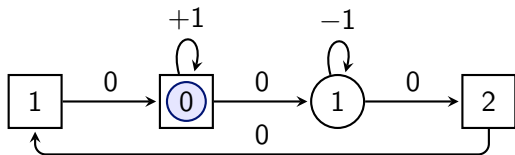
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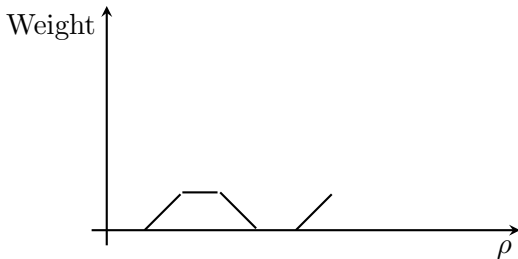
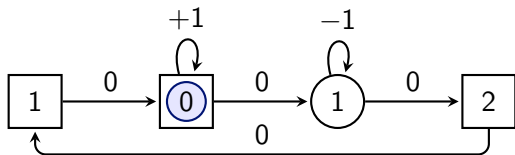
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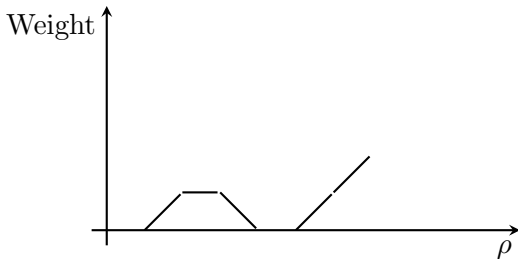
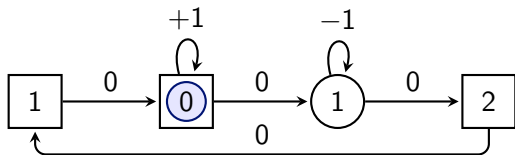
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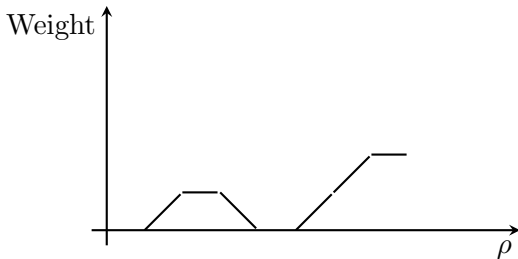
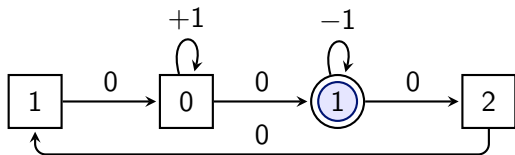
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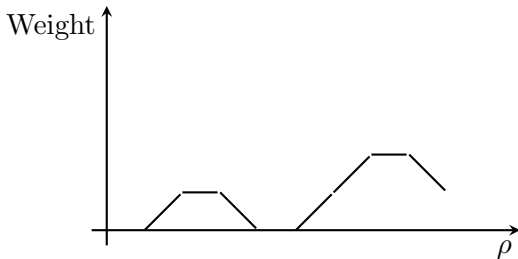
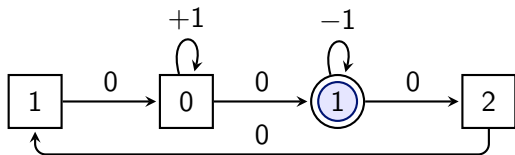
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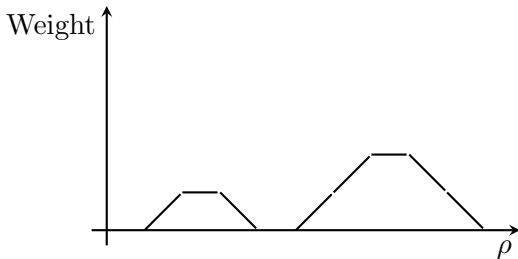
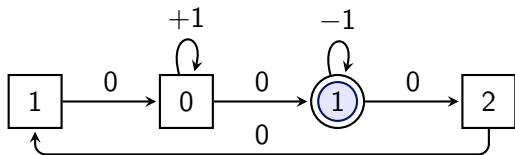
Cost Measure



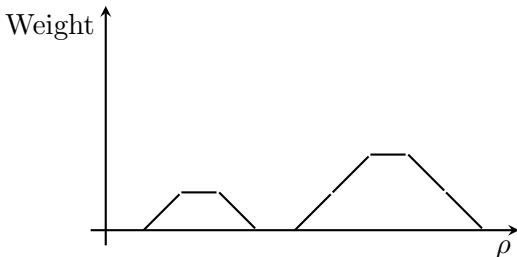
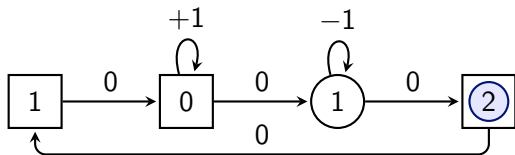
Cost Measure



Cost Measure



Cost Measure



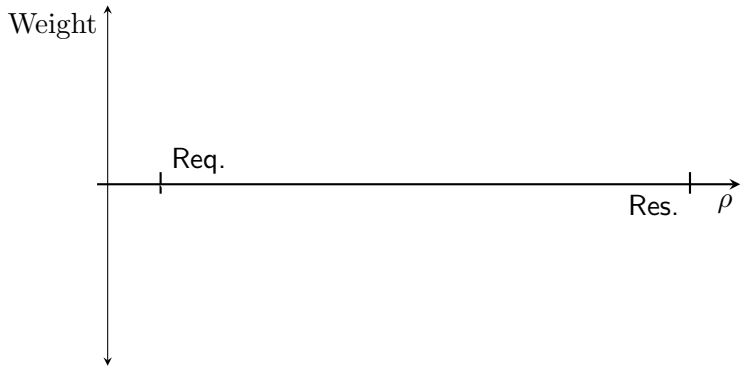
Intuitively:

Measure size of resource required to answer request

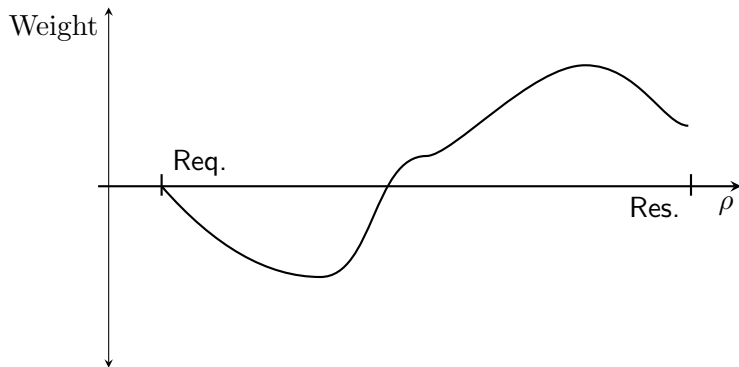
Cost Measure



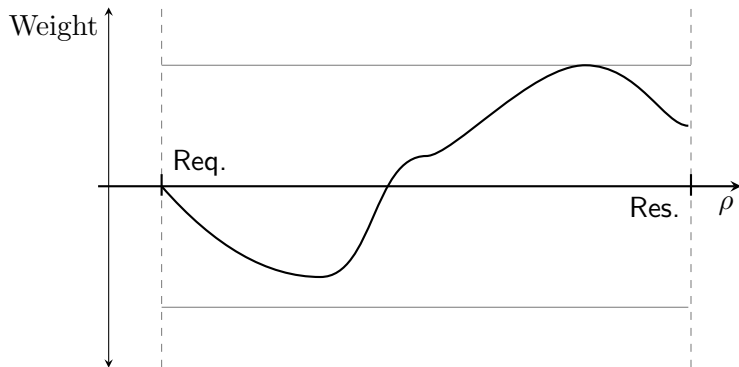
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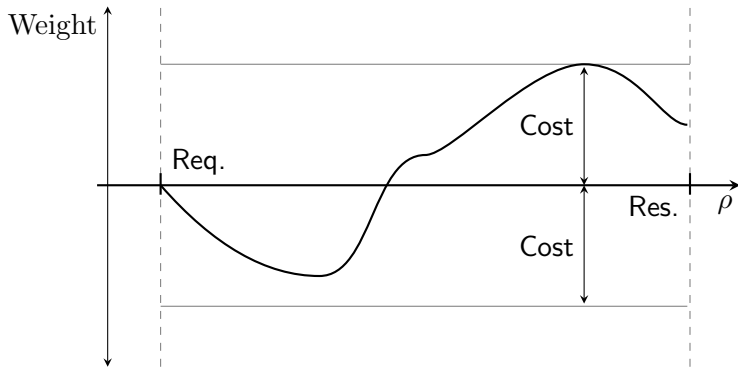
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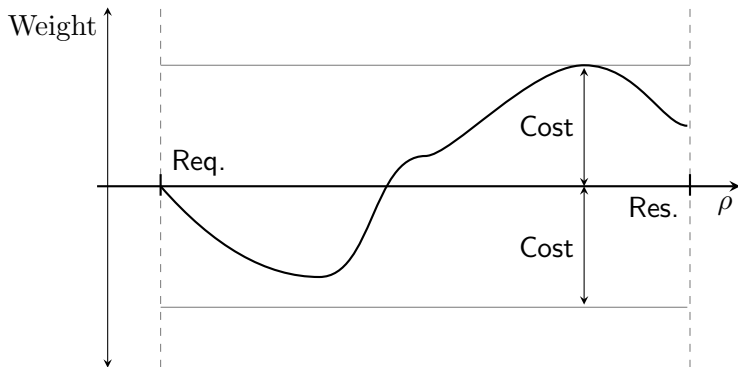
Cost Measure



Cost Measure

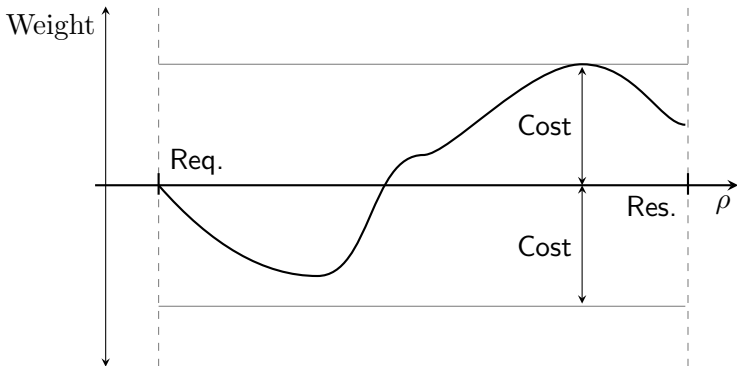


Cost Measure



Cost of answering: **Amplitude on path to answer**

Cost Measure



Cost of answering: **Amplitude on path to answer**

Goal for Player 0: Bound cost of almost all requests

Results

	Complexity	Memory	Bounds
Finitary	P _{TIME}	positional	n
Costs	UP \cap CO-UP	positional	nW

Results

	Complexity	Memory	Bounds
Finitary	P _{TIME}	positional	n
Costs	UP \cap CO-UP	positional	nW
Weights			

Results

	Complexity	Memory	Bounds
Finitary	P _{TIME}	positional	n
Costs	UP \cap co-UP	positional	nW
Weights	NP \cap co-NP		

Results

	Complexity	Memory	Bounds
Finitary	P TIME	positional	n
Costs	UP \cap CO-UP	positional	nW
Weights	NP \cap CO-NP	$\mathcal{O}(nd^2 W)$	

Results

	Complexity	Memory	Bounds
Finitary	P TIME	positional	n
Costs	UP \cap CO-UP	positional	nW
Weights	NP \cap CO-NP	$\mathcal{O}(nd^2 W)$	$\mathcal{O}((ndW)^2)$

Conclusion

Parity

–

Conclusion

Parity

–

Finitary

1

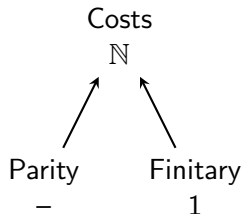
Conclusion

Costs
 \mathbb{N}

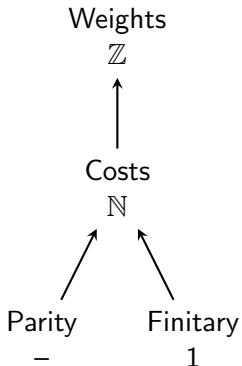
Parity
–

Finitary
1

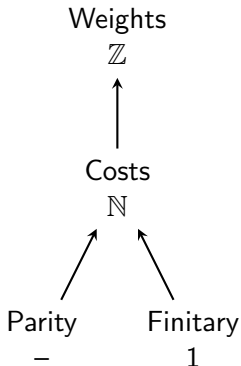
Conclusion



Conclusion



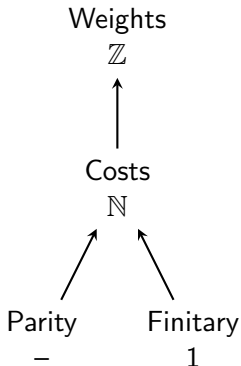
Conclusion



Take-Away: Greater expressiveness incurs

- small cost in terms of solving

Conclusion



Take-Away: Greater expressiveness incurs

- small cost in terms of solving
- great cost in size of strategy